# Did R&D Misallocation Contribute to Slower Growth?*


Nils H. Lehr†

*Boston University*


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**Abstract**

This paper provides evidence that rising frictions and their impact on the allocation of R&D resources have contributed to the slowdown in US productivity growth in recent decades. I develop a growth accounting framework allowing for rich firm heterogeneity in R&D productivity, exposure to frictions, as captured by an R&D wedge, and the rate at which private value created from innovation translates into growth. The model growth rate permits a closed-form decomposition into the frictionless level and an adjustment factor capturing the impact of frictions. I propose a methodology to measure the model primitives for a sample of US-listed firms from 1975 to 2014. Frictions can be measured using the R&D return, i.e., the ratio of value created from R&D to its cost, which I measure as the ratio of patent valuations to R&D expenditure over a 5-year window. I document large and persistent differences therein, which suggests significant frictions through the lens of the model. The evidence suggests financial frictions, adjustment costs and monopsony power over inventors as potential drivers of R&D return dispersion. Combining model and data, I estimate that frictions reduce economic growth by 18% on average and that their rise can account for 11% slower growth in 2000–14 or 30% of the overall observed slowdown. These findings are robust to a large set of alternative specifications and measurement error adjustments.



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†Email: nilslehr@bu.edu

# 1 Introduction

US productivity growth has slowed down significantly during the last two decades with similar developments in other developed nations. While productivity grew at a pace of 1.8% per year in 1949–1995, this rate declined to 1.2% in 2005–2018 (Aghion et al., 2023). This development is concerning as economists have long recognized productivity, or the rate at which an economy can create output from its production factors, as a key driver of prosperity and the economic success of developed nations (Solow, 1956).

In (semi-)endogenous growth theory, productivity grows as the product of innovation brought by research and development (R&D) investments (Romer, 1990; Aghion and Howitt, 1992; Jones, 1995). Slower growth can then be attributed to a decline in either R&D investments or the rate at which they translate into productivity growth, R&D productivity:

$$\text{Growth } (\downarrow) = \text{R\&D Investment } (-/\uparrow) \times \text{R\&D Productivity } (\downarrow).$$

Empirically, US R&D investment has remained stable or grown with 2.6% of GDP invested into R&D for 1970–95 compared to 2.85% for 2005–18 (FRED). Thus, this framework suggests that slower growth is driven by lower aggregate R&D productivity.

In this paper, I provide evidence that lower aggregate R&D productivity is partly due to rising misallocation in the R&D sector. While some firms appear to invest too much in R&D relative to the inventions they produce, others do too little, and increasingly so. Quantitatively, my estimates suggest this channel can account for an 11% lower growth rate, or around 30% of the overall growth slowdown documented above. Lower growth is, thus, not only driven by declining firm-level R&D productivity, as argued in Bloom et al. (2020), but also by a declining efficiency at which R&D resources are allocated among innovative firms.

I reach these conclusions based on a growth accounting framework nesting workhorse growth models. In the model, firms with potentially different *R&D productivity* hire R&D workers to maximize the private value created from innovation. I introduce frictions flexibly by allowing for exogenous *R&D wedges* in firms' first-order conditions that distort firms' demand for R&D inputs and which can be interpreted as implicit taxes on R&D expenditure. Growth occurs as a by-product of innovation, however, I allow for a potential gap between the private value created from innovation and its growth impact, which I refer to as *impact-value factor*. All else equal, firms with low impact-value factors conduct too much R&D from the perspective of growth maximization as they create a lot of private value, but little growth.

The model allows for a closed-form solution of the economic growth rate that can be decomposed into its frictionless level and an adjustment factor capturing resource misallocation due to R&D wedges, which I refer to as *R&D efficiency*. With a common impact-value factor, which is the baseline case in the literature, variation in R&D wedges reduces allocative efficiency by pushing the distribution of relative R&D efforts away from their growth-maximizing optimum—an R&D sector equivalent of Hsieh and Klenow (2009). Heterogeneity in impact-value factors can amplify, dampen, or even overturn this result. On the one hand, if firms with low impact-value factors also have low R&D wedges, then misallocation is even worse as R&D wedges push firms that already invest too much in R&D from a growth-maximizing perspective to do even more. On the other hand, the growth-maximizing R&D policy uses R&D wedges to offset differences in impact-value factors and maximize allocation efficiency. Thus, *R&D efficiency* provides a tool to quantify the impact of frictions on aggregate R&D productivity and economic growth.

I consider several extensions. First, my framework assumes that firms operate a single R&D production function. I show that my results equivalently apply in a framework with multi-research line firms as in Klette and Kortum (2004), however, the counterfactual holds constant the number of research lines across firms, which could be amplifying or dampening. Second, frictions are more costly under free entry of R&D firms as they tend to push up the wage of researchers and, thereby, reduce entry and the mass of research firms. Third, the formulae readily extend to the case in which R&D inputs are imperfect substitutes across firms. In this case, frictions tend to be less costly due to lower gains from input reallocation. Finally, my baseline framework assumes fixed R&D inputs at the aggregate level, but I show that the formulae extend to the case of a positive aggregate R&D supply elasticity. In this case, the level of frictions has a direct effect on growth, while it does not in the case of fixed supply of R&D inputs, in which only relative frictions matter.

Next, I propose a methodology to estimate the model primitives from firms' patents and financial statements and apply it to a sample of US–listed firms from 1975 to 2014. I measure firms' investment in innovation using R&D expenditure and the resulting private value created using patent valuations. Through the lens of the model, the ratio of value created and investment, which I refer to as *R&D return*, provides a direct measure of R&D wedges. I experiment with a range of proxies for impact-value factors based on theory-motivated profitability measures or citations. Together, these data provide me with the ingredients to estimate *R&D efficiency* on economic growth in my sample.

Before estimating the aggregate impact of R&D wedges, I investigate them at the micro-level. In a frictionless benchmark economy as in Romer (1990) or Aghion and Howitt (1992), firms equalize the marginal benefit to the marginal cost of R&D and, thereby, the R&D return as well. In contrast, I find large and highly persistent differences in measured R&D returns, and by extension R&D wedges, across firms and time. For example, the standard deviation of R&D returns is 42% larger than its counterpart for the return on capital, whose dispersion has been interpreted as a evidence for quantitatively important resource misallocation in the production sector (David et al., 2016). Notably, this dispersion is mostly among highly comparable firms, e.g., Qualcomm has a significantly larger R&D return compared to Intel. Quantitatively, 78% of the variation remains when focusing on differences among firms within 6-digit industry×year cells only. Finally, the strong persistence of R&D returns—with an implied annual auto-correlation coefficient around 0.9—suggest structural factors rather statistical noise. Taken at face value, the measured dispersion in R&D wedge suggests that friction play an important role in shaping the allocation of R&D resources in the economy—perhaps surprisingly, even among US-listed firms.

I perform numerous robustness exercises and find large measured R&D return dispersion throughout. First, following Bloom et al. (2020) I consider sales and employment growth as alternative measure of the private value of R&D output and find larger dispersion in measured R&D returns. Second, I follow complementary bootstrap and structural approaches to estimating the prevalence of measurement error and find no evidence for a strong measurement error component in R&D return dispersion. Lastly, I investigate a range of additional mechanisms directly, including the acquisition of innovative firms, fixed costs in R&D, knowledge capital, and alternative assumptions around the valuation of patents, and, again, find no evidence that they significantly contribute to measured R&D return dispersion. It, thus, appears that dispersion in measured R&D wedges is not driven by measurement details, suggesting potentially economic drivers.

R&D wedges are a direct measure of frictions in the model, however, I find that they are surprisingly hard to predict with empirical measures thereof. For example, they are uncorrelated with the return on capital, a common measure of investment frictions in physical capital, suggesting that both forms of investment are subject to independent sets of frictions. Similarly, I do not find any strong connection with proxies for R&D subsidies suggesting a limited role of government intervention in shaping R&D investments in my sample (Hsieh and Klenow, 2009). Only knowledge-intensity, Tobin's Q and R&D employment account for

5% or more of the variation in R&D wedges. The former two measures are potential proxies for financial frictions, which might be especially important in the case of R&D (Brown et al., 2009; Ewens et al., 2022). The positive correlation of R&D employment and wedges supports a framework in which monopsony power over inventors increases with their employment (Berger et al., 2022). Lastly, the relationship of R&D wedges and impact-value factors is ambiguous. While I find positive correlations using theory-based profitability or markup measures, I find negative correlations for proxies measuring growth impact with patent citations.

At the aggregate level, I estimate that growth is significantly slower due to low *R&D efficiency* and increasingly so. For the full sample, I estimate that economic growth is 18% lower due to R&D wedges, implying a frictionless annual growth rate of 1.9% against a realization of 1.5%. For comparison, Hsieh and Klenow (2009) estimate that US productivity would improve by 40% under the first-best allocation, while Berger et al. (2022) estimate a 21% output improvement in absence of monopsony. Naturally, achieving the frictionless growth rate might not be feasible in practice if R&D wedges are the product of technological or information frictions that cannot, or should not, be adjusted for.

Comparing the 1975–90 and 2000–14 period, I find that the evolution of R&D wedges alone yields an 11% lower growth rate in the latter period with associated welfare costs around 5%. According to these estimates, rising frictions can account for up to 30% of the $\frac{1.8\%-1.2\%}{1.8\%} \approx 33\%$ reduction in economic growth documented by Aghion et al. (2023). Thus, my estimates suggest that slower growth and lower R&D productivity is partly due to rising misallocation in the R&D sector with large adverse implication for welfare.

I investigate measurement concerns at the aggregate level in several robustness exercises. First, I show that my results are conservative when considering alternative measures for the private value of innovation. For example, the estimated *Impact of R&D wedges* increases from -18% to -25% (-40%) when measuring the private valuation of innovation using sales (employment) growth as in Bloom et al. (2020) instead of patent valuations. Second, I directly estimate the importance of idiosyncratic measurement error using bootstrapping and a structural composition. Perhaps surprisingly, I find that classical measurement error does not account for a significant share of the variation in measured R&D wedges and that accounting for it does not materially change the aggregate estimates. Finally, additional robustness across the measurement specification yields quantitatively similar estimates.

**Literature.** This paper contributes to three strands of the literature. First, I complement the growing literature investigating the origins of the recent decline in the economic growth rate by highlighting the importance of private frictions (Syverson, 2017). Similar to Akcigit and Ates (2021) and Olmstead-Rumsey (2022), I argue for declining aggregate R&D productivity as a core driver, however, I attribute this change to rising misallocation instead of declining micro-level R&D productivity or knowledge spillovers.[1] This perspective is similar to de Ridder (2023), Aghion et al. (2023) and Ayerst (2022), who propose models in which rising heterogeneity in impact-value factors—in form of differences in static production efficiency, markups, and knowledge spillovers—curtailed economic growth by increasing R&D misallocation. Instead, I focus on the contribution of private frictions, as captured by R&D wedges, and follow a sufficient statistic approach allowing for a direct mapping between data and model, rather than structural estimation. I estimate that rising frictions can account for 30% of the growth slowdown documented in the literature.

Second, I provide a new framework to investigate the drivers of aggregate R&D resource allocation and productivity. The early endogenous growth literature first identified innovation as the main driving force behind economic growth and highlighted the potential for under- or over-provision of innovation due to externalities (Romer, 1990; Aghion and Howitt, 1992). More recent contributions have focused on the allocation of R&D resources across firms, which might be inefficient under the presence of heterogeneity in spillovers or firms' ability to benefit from inventions of a given quality (de Ridder, 2023; Aghion et al., 2022; Akcigit et al., 2022). For example, heterogeneity firms' ability to enforce patents can lead firms with low enforcement capabilities to under-invest in innovation (Mezzanotti, 2021). Similarly, firms might erect entry barriers through defensive innovation without creating productivity improvements (Manera, 2022). My framework is closely connected, but differs along several dimensions. First, I allow for private frictions and estimate that they have a significant impact on economic growth. Second, I develop a closed-form growth rate decomposition clarifying how frictions and impact-value factors shape economic growth. Importantly, both factors can offset or amplify each other and, thus, require a joint treatment. Third, my framework aims to be comprehensive across mechanisms by measuring reduced form wedges in the data. At its best, this approach can take into account a range of economic forces simultaneously, however, it cannot isolate individual channels without further information.

[1] Bloom et al. (2020) also argue that aggregate R&D productivity has declined, however, their focus is a long-run, steady decline in R&D productivity as "ideas are getting harder to find," in line with the predictions of semi-endogenous growth theory (Jones, 1995).

Third, I contribute to the literature on factor misallocation by proving evidence on its pervasiveness in the R&D sector as captured by R&D return dispersion. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) first argued that capital misallocation across firms, as captured by dispersion in the return on capital, significantly reduces aggregate productivity in the US and can explain a significant share of the productivity gap between developed and developing countries. The subsequent literature investigated potential drivers of dispersion in the return on capital, including government intervention and financial, adjustment, and information frictions, and found that it is surprisingly hard to attribute to individual mechanisms, which I also find for R&D returns (Asker et al., 2014; Midrigan and Xu, 2014; David et al., 2016; David and Venkateswaran, 2019). I complement the literature by focusing on R&D investments instead of static production factors, which introduces a dynamic component linking factor return heterogeneity to the growth rate instead of static production efficiency. Similar to the results in Hsieh and Klenow (2009) for the production sector, my framework allows for a closed-form solution and direct link to the data. My estimates suggest that US productivity growth it 21% lower due to resource misallocation, while Hsieh and Klenow (2009) estimate a 30%–40% lower productivity level for the US compared to an efficient allocation. Perhaps surprisingly, I show that the sources of frictions appear to be different. For example, my measure of frictions, the R&D return, is uncorrelated with the return on capital, which is often considered a summary measure of frictions in the production sector. I also find mixed evidence on the contribution of channels discussed prominently in the literature such as government subsidies or financial frictions. In contrast, König et al. (2022) find that product market frictions led to an inefficient R&D allocation in China. Similarly, financial frictions are often considered to be particularly severe in the case of intangible investments, including R&D (Brown et al., 2009). My focus on US–listed firms and differences in institutional context appear to be a likely explanation of the contrasting findings. The strongest predictor of frictions in my sample is R&D employment, which could be rationalized, e.g., by increasing monopsony power exerted by large employers (Berger et al., 2022). Such a channel could also explain rising frictions in light rising overall concentration documented, e.g., in Autor et al. (2020).

**Organization.** Section 2 presents the model and derives the main formulae. Section 3 discusses data and measurement. Section 4 establishes stylized facts for the measured frictions and investigates potential underlying mechanisms. Section 5 combines data and theory to investigate the impact of frictions on economic growth, while Section 6 concludes.

# 2 Theory

This section introduces a heterogeneous firms growth model allowing flexibly for frictions in firms' input choices. The framework is sufficiently general as to allow for interpretations in the spirit of alternative growth theory traditions including Romer (1990), Aghion and Howitt (1992) or Acemoglu and Cao (2015). The model's equilibrium growth rate can be derived in closed form and allows for a direct assessment of the growth impact of frictions. The formulae derived in this section are at the core of the subsequent empirical analysis.

## 2.1 Model Setup

Time is infinite, discrete, and indexed by $t$.

**Production.** Output is the product of productivity $A_t$ and production labor input $L_t \cdot \tilde{L}$:

$$Y_t = A_t \cdot L_t \cdot \tilde{L}, \tag{1}$$

Productivity encompasses technological efficiency and static production frictions such as markups.[2] I focus on its evolution based on changes in technological efficiency only.

**Firms.** There is a unit mass of innovative firms indexed by $i$ hiring R&D input $\ell_{it}$ at input price $W_t$ to achieve mass $z_{it}$ of innovations:[3]

$$z_{it} = \varphi_{it} \cdot \ell_{it}^{\gamma} \quad \text{with} \quad 0 < \gamma < 1. \tag{2}$$

Firms assign value $V_{it}$ to innovations, which I take as given. In workhorse growth models, this value is linked to resulting profits and innovation opportunities (Gancia and Zilibotti, 2005; Aghion et al., 2014).[4] Firms are subject to R&D wedge $\Delta_{it}$ such that their equilibrium R&D input choice $\ell_{it}^*$ satisfies

$$\left.\frac{\partial z_{it}}{\partial \ell_{it}}\right|_{\ell_{it}=\ell_{it}^*} \cdot V_{it} = (1 + \Delta_{it}) \cdot W_t. \tag{3}$$

The left-hand side is the marginal benefit of research input, while the right-hand side is the marginal cost adjusted for the R&D wedge. If $\Delta_{it} = 0$, we recover the frictionless benchmark

[2] Peters (2020) decomposes productivity into markup heterogeneity and technological efficiency. My results are compatible as long as markup heterogeneity is constant and independent of frictions.

[3] Alternatively, $z_{it}$ can be interpreted as the arrival rate of inventions in a continuous time setup.

[4] I provide examples of $V_{it}$ for alternative microfoundations in Appendix G.

in which firms equalize marginal benefit and cost. Otherwise, firms' choices are distorted relative to the benchmark with larger wedges resulting in lower demand for R&D inputs.

In theory, there could be many potential mechanisms driving variation in R&D wedges $\Delta_{it}$ across firms including, but not limited to, financial frictions, adjustment costs or capacity constraints, market power in the R&D input market, and R&D subsidies. For example, high R&D wedges can capture constraints on firms' choice of R&D inputs due to financial frictions or adjustment costs (Midrigan and Xu, 2014; Asker et al., 2014). Similarly, low R&D wedges can capture R&D subsidies, which reduce firms' marginal cost below the market price (Hsieh and Klenow, 2009). I discuss these and other mechanisms in Appendix G and take R&D wedges as given here. Naturally, the nature of R&D wedges in practice determines whether we should interpret them as technological facts or as objects subject to economic policy.

**Factor markets.** I assume that aggregate R&D input $L_t$ is exogenous, capturing the idea that research talent is scarce and supplied inelastically (Goolsbee, 2003; Wilson, 2009):

$$L_t = \int_0^1 \ell_{it} \cdot di. \tag{4}$$

Mechanically, this assumption emphasizes the allocation of input factors within the R&D sector instead of across the production and R&D sector. An alternative interpretation is that R&D policy already fixes the size of the R&D sector at the optimal level, such that the allocation of resources within it remains the relevant margin of concern.[5]

**Growth.** Economic growth occurs as the byproduct of innovation and is the aggregate of the probability that inventions occur times their growth impact. The latter is linked to an invention's value to the firm via the impact-value factor $\zeta_{it}$, which acts as an exchange rate between both concepts. Firms with a large impact-value factor contribute more productivity growth per dollar of private value created. The growth rate is given by

$$g_t \equiv \frac{A_{t+1} - A_t}{A_t} = A_t^{-\phi} \cdot \int_0^1 \zeta_{it} \cdot z_{it} \cdot V_{it} \cdot di, \tag{5}$$

where $\phi \geq$ measures the strength of the "phishing-out" effect that is necessary to achieve balanced growth in a semi-endogenous growth framework (Jones, 1995).

[5]Note, also, that this assumption implicitly rules out any direct waste of R&D resources linked to R&D wedges, e.g., due to "real" adjustment cost. Any adjustment costs, thus, should take the form of either production labor or cash payments. This setup can capture, for example, hiring and recruiting costs, but fails to capture concerns such as lower productivity of newly hired R&D workers or temporary skill mismatch.

Impact-value factor $\zeta_{it}$ has a prominent role in the growth literature as it determines the degree to which firms' incentives are aligned with a growth-maximizing planner. The early endogenous growth literature emphasizes that firms might not be able to appropriate the full value generated from their innovation to society, implying that social value exceeds private value or $\zeta_{it} > 1$ (Romer, 1986). On the other hand, the Neo-Schumpeterian literature argues that the business stealing effect acts as a counterbalancing force as firms do not take into account the economic damage imposed on firms that are made obsolete by competitor innovation (Aghion and Howitt, 1992). In recent contributions, differences in impact-value factors occur as some firms are able to earn larger profits from a given invention, better at protecting their intellectual property, or less prone to be replaced by challengers (Akcigit and Ates, 2021; de Ridder, 2023; Mezzanotti, 2021; Aghion et al., 2022; König et al., 2022; Manera, 2022; Olmstead-Rumsey, 2022). I discuss these microfoundations further in Appendix G and, as in the case of R&D wedges, take impact-value factors as given here.[6]

This discussion also highlights that the relationship between R&D wedges and impact value factors, to the degree that there is heterogeneity in either, is not clear ex-ante. On the one hand, one could imagine that particularly financially constrained firms, such as startups, are also less able to extract rents from their inventions, suggesting a positive relationship. On the other hand, one could imagine that dominant firms have some degree of market power over R&D inputs, while also earning larger profits from a given invention due to their existing distribution channels or economies of scale, suggesting a negative relationship. Thus, it is an empirical question how wedges and factors are systematically related, if at all.

**Consumer Welfare.** For welfare calculations, I assume that households have discount factor $\beta$ and CRRA utility over per-capita consumption, which equals output in equilibrium:

$$\mathcal{W}(\{Y_{t+\tau}\}) = \sum_{\tau=0}^{\infty} \beta^{\tau} \cdot \frac{(Y_{t+\tau}/(L_{t+\tau} \cdot (1+\tilde{L}))^{1-\sigma} - 1}{1-\sigma}. \tag{6}$$

This formulation ignores the quantity effect of a growing population over time and, thereby, allows for a more direct comparison across economies with and without population growth.

[6] Another sources of divergence between private and planner valuation of innovation are knowledge externalities. For example, Akcigit and Kerr (2018) propose a model in which some inventions improve the productivity of follow-on inventions by competitors. Such inventions are high value from the perspective of a planner, but might be low value from the perspective of the firm due to the heightened risk of replacement by a competitor. Within my framework I will ignore the possibility of such knowledge externalities due to data limitations. See Bloom et al. (2013) for an empirical investigation of knowledge spillovers.

**Equilibrium.** I use two simplified equilibrium definitions in deriving the main results. The Competitive Equilibrium respects the equation detailed above.

**Definition 1.** *A Competitive Equilibrium is a sequence* $\{\{V_{it}, \varphi_{it}, \Delta_{it}, \zeta_{it}, \ell_{it}\}_{i\in[0,1]}, W_t, g_t, Y_t\}_{t=0,\dots,\infty}$ *for a given* $Y_0$ *and* $\{L_t\}_{t=0,\dots,\infty}$ *satisfying equations (1)-(5).*

The Planner Equilibrium instead allocates R&D inputs to maximize the growth rate.

**Definition 2.** *A Planner Equilibrium is a sequence* $\{\{V_{it}, \varphi_{it}, \zeta_{it}, \ell_{it}\}_{i\in[0,1]}, g_t, Y_t\}_{t=0,\dots,\infty}$ *for a given* $Y_0$ *and* $\{L_t\}_{t=0,\dots,\infty}$ *maximizing economic growth* $\{g_t\}_{t=0,\dots,\infty}$ *and satisfying equations (1), (2), (4), and (5).*

## 2.2 Results

With the model in place, we can characterize the equilibrium economic growth rate in closed form and decompose it into three terms as shown in Proposition 1. The first two terms jointly characterize the economic growth rate in a competitive equilibrium in absence of, or with homogeneous, R&D wedges, which I denote by $g_t^C$. I discuss the economic interpretation of both components below when considering optimal R&D policy. The third term, which I refer to as *R&D efficiency*, captures the impact of R&D wedges on economic growth.[7] R&D efficiency depends on the distribution of R&D wedges, but not their average level. Intuitively, any aggregate excess or insufficient demand for R&D resources is balanced by the R&D input price due to the fixed aggregate supply of thereof and, thus, does not have a direct impact on economic growth.

**Proposition 1.** *Under equations (2)-(5), we can express the economic growth rate in a Competitive Growth Equilibrium as the product of three terms:*

$$g_t = \underbrace{\frac{L_t^{\gamma}}{A_t^{\phi}} \cdot \left(\int_0^1 (\theta_{it} \cdot \zeta_{it})^{\frac{1}{1-\gamma}}\, di\right)^{1-\gamma}}_{=\ \textit{Frontier Growth Rate}\ g_t^F} \cdot \underbrace{\left(\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it}^{\frac{1}{1-\gamma}}\, di\right)^{\gamma-1}}_{\equiv\ \textit{Policy Opportunity}\ \Lambda_t} \cdot \underbrace{\frac{\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}} di}{\left(\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} di\right)^{\gamma}}}_{\equiv\ \textit{R\&D efficiency}\ \Xi_t}, \quad (7)$$

*where* $\tilde{\zeta}_{it} \equiv \zeta_{it} / \left(\int_0^1 \omega_{it} \cdot \zeta_{it} di\right)$ *and* $\omega_{it} \equiv \theta_{it}^{\frac{1}{1-\gamma}} / \left(\int_0^1 \theta_{it}^{\frac{1}{1-\gamma}} di\right)$ *are the normalized impact-value factor and an R&D productivity weight, respectively, with R&D productivity* $\theta_{it} \equiv \varphi_{it} \cdot V_{it}$.

[7] The word "efficiency" is an imperfect choice here, since full efficiency might not be achievable, e.g., due to adjustment frictions. Furthermore, depending on the impact-value factor, there are scenarios with above 100% efficiency as explained above. Nonetheless, it captures the nature of the term as a deviation from the frictionless benchmark.

To get a better understanding of R&D efficiency, it is useful to first consider the case where impact-value factors are constant or, alternatively, independent of R&D wedges. We, thus, assume that particularly constrained firms do not systematically create more or less growth impact per private value created. In this case, dispersion in R&D wedges strictly reduces economic growth as shown in Corollary 1. Intuitively, dispersion in R&D wedges leads to misallocation of R&D resources as firms with high wedges hire too few R&D workers and vice versa. Resultingly, we have a growth-model equivalent to the impact of "output-wedges" on aggregate productivity in Hsieh and Klenow (2009).

**Corollary 1.** *Suppose $\zeta_{it}$ is constant or independent of $(1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}}$, then the R&D efficiency depends on the joint distribution of R&D wedges and productivity only:*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}} di}{\left(\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} di\right)^{\gamma}}. \tag{8}$$

*Up to a $2^{nd}$-order approximation, R&D efficiency is strictly decreasing in their dispersion and achieves a maximum of 1 if R&D wedges are equalized.*

The analysis becomes slightly more complex once we relax the assumption of orthogonal impact-value factors as highlighted in Proposition 2. As long as R&D wedges and impact-value factors have a positive, or at worst weakly negative, correlation, we can understand them as either amplifying or dampening R&D wedges. In particular, if there is positive correlation, i.e., if particularly constrained firms also achieve a high growth impact per dollar of private value created, then impact-value factors amplify the misallocation created from R&D wedges. Conversely, they dampen their effect in the case of (weak) negative correlation, i.e., if constrained firms have lower growth impact per private value created.

**Proposition 2.** *Suppose that the $\omega_{it}$–weighted covariance of log R&D wedges and impact-value factors, $\sigma_{\Delta,\zeta}$, is weakly positive or larger in absolute value than half the $\omega_{it}$–weighted variance of log R&D wedges, $\sigma^2_{\Delta}$. Then, up to a $2^{nd}$-order approximation, the Impact of R&D wedges can be expressed as*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}\cdot\tilde{\beta}} di}{\left(\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}\cdot\tilde{\beta}} di\right)^{-\gamma}}, \tag{9}$$

*where $\tilde{\beta} = \sqrt{1+2\cdot\beta}$ is an adjustment factor depending on $\beta \equiv \frac{\sigma_{\delta,\zeta}}{\sigma^2_{\delta}}$, which is also the weighted OLS coefficient when regressing impact-value factors on R&D wedges in logs.*

A couple of examples highlight the underlying economics. Consider a world in which young firms are financially constrained and less skilled in extracting rents from their inventions, e.g., because of fewer legal resources to defend their patents. Thus, young firms have large R&D wedges, because of financial frictions, and high impact-value factors. Then, R&D wedges are even more costly for growth since such firms would have invested insufficiently in R&D even without wedges, but are now pushed even further away from the optimal R&D employment.

Alternatively, consider a world in which large firms have market power over inventors, but are also more skilled in extracting rents from their inventions. In this case, we have that firms with high R&D wedges, due to monopsony power, tend to have lower impact-value factors, and, as a result, market power pushes the allocation of R&D workers away from low impact-value factor firms and, thus, is less costly in terms of economic growth.

In summary, Proposition 2 allows us to estimate the impact of R&D wedges on the allocation of R&D resources and economic growth as long as we have measures of $\{\gamma, \{\omega_{it}, \Delta_{it}, \zeta_{it}\}, \tilde{\beta}_t\}$, which is the subject of the next section.

Before concluding this section, I want to briefly discuss optimal growth policy, which I derive in Proposition 3. To achieve maximal growth, the planner introduces a perfect negative correlation of R&D wedges and impact-value factors by using the subsidy component of R&D wedges to offset any frictions and heterogeneity in impact-value factors. This allocation achieves the frontier growth rate by setting $\Lambda_t \cdot \Xi_t = 1$, which suggests that any $\Lambda_t \cdot \Xi_t < 1$ yields a growth rate within the economy's growth possibility frontier. The key intuition is that firms maximize value, while the planner wants to maximize growth. As long as both are not perfectly aligned, the allocation of R&D inputs is inefficient from the perspective of growth maximization. This case is at the heart of the recent endogenous growth literature. For example, in the models proposed in de Ridder (2023) and Aghion et al. (2022), firms have heterogeneous markups unrelated to the productivity impact of their invention. Hence, firms with high exogenous markups earn larger profits on a given innovation and, thus, have lower impact-value factors. Differences in these exogenous markups then lead to dispersion in impact-value factors, which can reduce growth and opens up the gap between frontier and competitive growth rate.

**Proposition 3.** *Let $g_t^*$ be the growth frontier achieved in the Planner Growth Equilibrium, then $g_t^* = g_t^F$. Furthermore, this allocation can be achieved by setting the R&D subsidy component of $\Delta_{it}$ such that $\zeta_{it} \cdot (1 + \Delta_{it})$ is constant across firms, which achieves $\Lambda_t \cdot \Xi_t = 1$.*

## 2.3 Extensions

I consider several model extensions in Appendix A.2, which I briefly highlight here.

**Free entry.** The model assumes that the mass of innovative firms is fixed, however, changes in the economic environment might affect firms' incentives to enter and exit the economy. I show that allowing for entry can amplify the cost of private frictions. R&D wedges tend to reduce the expected profits of innovative firms due to rising input costs, resulting in lower entry. Fewer firms implies more researchers per firm, which reduces their productivity due to decreasing returns to scale and, thus, lowers growth. Importantly, the impact of frictions on entry is independent of firms' impact-value factors, such that policies improving the allocation of R&D across firms may reduce entry vis-à-vis a competitive equilibrium.

**Multi-research line firms.** A long tradition in endogenous growth has modeled the innovation sector with multi-research line firms (Klette and Kortum, 2004). The distribution of research lines is an endogenous object in this class of models, driven by firms' innovation. I show that the formulae developed above extend to this alternative framework, however, the counterfactual holds constant the distribution of research lines across firms. The estimated growth impacts are conservative if firms that expand in absence of frictions also tend to be more productive.

**Specialization in input markets.** The recent literature on labor market power emphasizes that firms might be imperfect substitutes from the perspective of workers due to amenities or specialization (Card et al., 2018; Berger et al., 2022). Thus, gains from input reallocation across firms might be limited and firms might have market power over inputs. I show that the formulae developed above are preserved under this scenario with a lower implied scale elasticity $\gamma$, which decreases in the degree to which inputs are specialized. Resultingly, R&D wedges tend to be less costly due to lower gains from reallocation.

**Abundant resources.** The model assumes fixed aggregate R&D inputs, implying that the level of R&D wedges does not affect growth. I show that the formulae derived above extend to the case with positive input supply elasticity, however, there is a supply adjustment term depending on the average R&D wedge. The gains from reallocation thus coincide as long as this average remains constant. The formulae also extend to multiple R&D production factors as long as their supply is perfectly inelastic and frictions are common at the firm-level.

# 3 Data and Measurement

## 3.1 Data

I focus my empirical analysis on research-active firms listed on US stock exchanges. I choose this sample as there is sufficient data available to measure the model primitives and directly apply the formulae developed above. The proposed measurement approach requires three pieces of information on R&D: expenditure, value created, and growth impact.

I obtain annual firm-level R&D expenditure from WRDS Compustat, which collects the information from mandatory filings. This data also reports firms' industry classification and additional accounting data including annual sales and employment.

I use patents to measure the private value created from R&D and its growth impact. Patents are arguably the most direct measure of R&D output available to researchers. They capture an invention that the issuing patent office, here the US Patent and Trademark Office (USPTO), deemed both new and useful. Patents grant the owners exclusive rights to the use of inventions described therein, giving firms strong incentives to patent. Nonetheless, using patents might yield an incomplete picture as not all inventions are patented (Cohen et al., 2000). I propose to address this concern by focusing on firms that tend to use patents and by investigating robustness using measures independent of the patent system.

I use patent valuation estimates from Kogan et al. (2017) to measure the private value created from innovation. Their methodology uses the firm's stock returns around the patent announcement to estimate its value such that larger returns are translated into higher valuations. Patent valuations directly capture the private value of an invention, which is directly linked to firms' incentives to innovate. In contrast, other patent-based measures of innovation, such as raw counts or citations received, capture the quantity of innovation, but not its value to the firm. As discussed in the previous section, divergence between the two concepts is an important object of interest when estimating the aggregate impact of private frictions.

I consider forward-citations as a potential measure of the growth impact of R&D as I discuss in detail below. I construct forward-citations, i.e., citations received, by the patent within the first 5 years since the patent grant using the USPTO Patentsview's citations files. Limiting the time-window ensures that patents granted earlier do not mechanically receive more citations. I normalize this measure by the average forward-citations within an application year to make the measure comparable across years (Kogan et al., 2017).

I aggregate citations and patent valuation to the firm-year-level using the patent-to-firm mapping developed in Kogan et al. (2017). Patents are recorded in their application year to reflect the timing of innovation. The final dataset has annual observations of firm-level R&D expenditure, patent valuations, and forward-citations.

I restrict the sample to 1975-2014 and drop firms with consistently low R&D expenditure (less than 2.5m 2012 USD per year), low patenting (less than 2.5 patents per year) or less than 5 years in sample. The final sample covers more than 80% of R&D expenditure in Compustat and patent valuations in Kogan et al. (2017) for the 1975-2014 period as well as 40% of the R&D recorded in BEA accounts. See Appendix B for further data details.

## 3.2 Measurement

I propose a straight-forward approach to measuring the model primitives. First, there is a consensus in the literature on setting $\gamma = 1/2$, which implies an elasticity of R&D expenditure to unit cost of -1 (Acemoglu et al., 2018; Akcigit and Kerr, 2018).

Second, R&D wedges can be measured up to a constant factor directly from the average R&D return, i.e., the ratio of value created from R&D divided by its cost:

$$\frac{z_{it}V_{it}}{W_t \ell_{it}} = \frac{1}{\gamma} \cdot (1 + \Delta_{it}). \tag{10}$$

Through the lens of the model, firms with high R&D returns appear more constrained, implying larger wedges. Key for this interpretation are common, log-linear production and cost functions, which yield proportional marginal and average returns and are standard in the literature (Gancia and Zilibotti, 2005; Aghion et al., 2014).[8] A similar measurement approach is pursued in Hsieh and Klenow (2009) for production wedges.

I implement the measurement using 5-year windows with a 1-year lag between R&D expenditure and patent valuations:

$$\widehat{1 + \Delta_{it}} = \gamma \cdot \frac{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it+s}}{\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s}}. \tag{11}$$

Note that the relevant formulae are HD(0) in $1 + \Delta_{it}$ such that any constant factor $\gamma$ has no bearing on the aggregate measures.

[8] In a more general setup, the equilibrium average R&D return is the ratio of marginal scale elasticities for the cost and R&D production function. In models where these are constants, i.e. where both functions take the form of $A \cdot \ell^B$ with $B$ being a parameter, the average R&D return is thus a constant.

Three measurement concerns emerge immediately. First, R&D return dispersion may arise due to differences in the scale elasticity of the innovation production function $\gamma$ across firms. Without adjustment, one would confound those for differences in R&D wedges. I, thus, residualize measured R&D wedges with respect to industry $\times$ year cells under the assumption that technologies are similar within industries. Secondly, accurate measurement of R&D wedges requires ex-ante expected R&D returns as firms equalize expected marginal benefits to marginal costs.[9] However, realized R&D returns might differ from their expected value due to the uncertainty inherent in the innovation process (Akcigit and Kerr, 2018). I, thus, restrict the sample to observations with at least 50 patents to leverage the power of the law of large numbers in closing the gap between averages and expectations. This approach does not safeguard against ex-post firm-level shocks that yield common variation in realized patent valuations and which I investigate separately. Finally, not all inventions are patented and the share of inventions patented might differ across firms (Cohen et al., 2000). Such differences may lead to variation in measured R&D returns due to patenting choices rather than R&D wedges. Following Bloom et al. (2020), I use non-negative changes in sales, employment, or the ratio thereof as alternative measures of innovation output following the idea that successful inventions lead firms to expand. My alternative measure of R&D wedges is then

$$\widehat{1+\Delta_{it}} = \gamma \cdot \frac{\sum_{s=0}^{4} \max\{\ X_{it+s} - X_{it+s-1}, 0\}}{\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s}} \quad \text{with} \quad X \in \{\texttt{Sales},\ \texttt{Empl.}, \frac{\texttt{Sales}}{\texttt{Empl.}}\}. \tag{12}$$

I discuss further concerns in conjunction with my main findings below and in Section 5.3.

Third, R&D productivity can be measured from the firms' first-order conditions as

$$\theta_{it} = (1+\Delta_{it}) \times (W_t \cdot \ell_{it})^{1-\gamma} \cdot W_t^{\gamma}. \tag{13}$$

Note, again, that the formulae developed above are scale independent such that I can drop the common wage intercept.[10] I, thus, measure R&D productivity as

$$\hat{\theta}_{it} = (\widehat{1+\Delta_{it}}) \cdot \left( \sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s} \right)^{1-\gamma}. \tag{14}$$

[9]The model Section 2 assumed that inventions are realized instantaneously and without uncertainty about the value of inventions $V_{it}$ or the mass of inventions $z_{it}$. More generally, the appropriate value is the expected discounted value created from inventions, which is proportional to the expected value created assuming constant discount rates across firms and a common gap between investment and realization.

[10]If wages are heterogeneous across firms, then my measurement of $\theta_{it}$ captures productivity and wage heterogeneity. Unfortunately, my data does not allow me to directly adjust for price differences in R&D inputs across firms.

Finally, I consider three approaches to measuring the impact-value factor and estimating its relationship to R&D wedges. In the first approach, I follow the workhorse growth models and assume it constant across firms. In the second approach, I use markup measures guided by the result that the impact-value factor is directly linked to them in a limit-pricing setup. Firms with high quality innovation have large markups, but are also not fully able to capture the additional social value created by the higher innovation quality.[11] I measure markups either using the estimates in Loecker et al. (2020) or, alternatively, via the value implied by firms' profit rates. The impact-value factor is then given by

$$\hat{\zeta}_{it} = \hat{\mu}_{it} \quad \text{or} \quad \hat{\zeta}_{it} = \frac{\sum_{s=0}^{4} \texttt{Sales}_{it}}{\sum_{s=0}^{4} \texttt{Sales}_{it} - \sum_{s=0}^{4} \texttt{Profit}_{it}}. \tag{15}$$

In the third approach, I propose a direct measure of the growth-impact of inventions and use it to measure the impact-value factor. A natural starting point is the ratio of patent citations to valuations, which would be accurate if citations measure the growth impact of an invention up to a constant factor:[12]

$$\hat{\zeta}_{it} = \frac{\sum_{s=0}^{4} \texttt{Patent Citations}_{it+s}}{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it-1+s}}. \tag{16}$$

One potential concern with this measure is heterogeneity in citation conventions across industries or time that affect the relative frequency of citations even if growth impacts are comparable.[13] I thus residualize measured impact-value factors with respect to industry×year fixed effects. Another, potentially more pressing, concern when relating impact-value factor to the R&D return is that they might be related by construction due to the use of patent valuations. I, thus, replace patent valuations with changes in sales whenever I directly relate impact-value factor to R&D wedges. I also present robustness around the measurement choice for growth impact using the patent impact measure developed in Kelly et al. (2021), which uses natural language processing to measure whether concepts developed in a patent are subsequently adopted in new patents.

[11] In particular, let $\lambda_i > 1$ be the quality improvement over a potential competitor in a model with limit pricing. Then, profits are proportional to markup $1 - \lambda_i^{-1}$, however, the growth impact is proportional to $\lambda_i - 1$. The impact-value factor is the ratio of both, which is proportional to $\lambda_i$ itself. Thus, we can use either markups or profit rates to back-out the implied $\lambda_i$.

[12] Ayerst (2022) employs this ratio with a similar interpretation. See Akcigit and Kerr (2018) for a similar interpretation of patent citations.

[13] For example, citation counts likely depend on the degree to which innovation is cumulative, as, e.g., in semi-conductors, rather than more independent of each other, as for molecules in drugs.

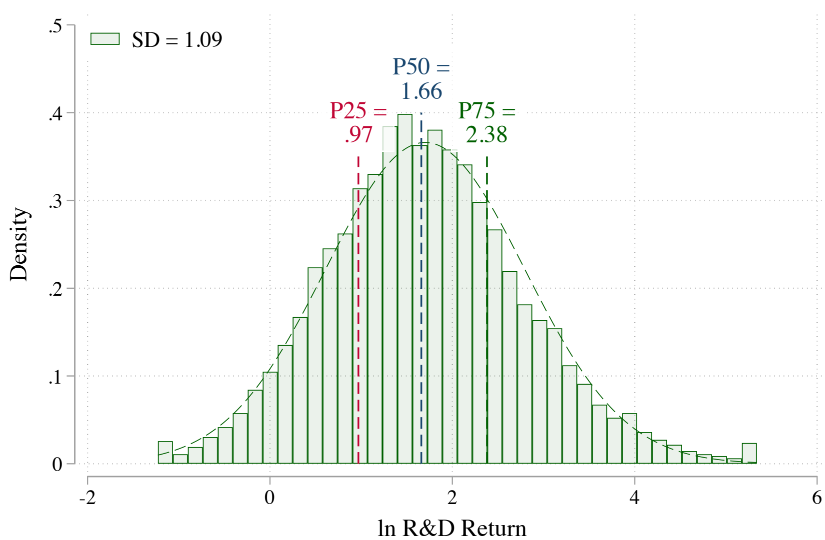
# 4 Exploring R&D Returns

Before estimating aggregate measures, I investigate the behavior of R&D wedges and their relationship to impact-value factors in the data. I use the terms R&D wedges and R&D returns interchangeably here given their measurement equivalence in my context.

## 4.1 Exploring R&D Returns

In the frictionless benchmark economy without subsidies, R&D wedges are equalized across firms and, thus, measured R&D returns should be as well. Instead, I find large dispersion in measured R&D returns as highlighted by their histogram plotted in Figure 1. A firm at the 75th percentile of the distribution has twice the median return with a similar gap between the median and 25th percentile.[14] The standard deviation of log R&D returns is 1.1.[15]

Figure 1: R&D Returns are Highly Dispersed




*Notes*: Figure plots the histogram of the log average R&D returns and density function of a normal distribution with same mean and variance. R&D returns are measured as the 5-year total patent valuation divided by 5-year R&D expenditures lagged by one year. See Section 3 and Appendix B for data details.

Furthermore, R&D return dispersion has increased throughout the sample. Comparing the early and late sample in Panel B of Table 1, I find that R&D return dispersion has risen by 32%. Rising dispersion is a broad phenomenon with about 64% of industries having more dispersed R&D returns in the later sample as reported in Appendix C.4.

[14] $\exp(2.38 - 1.66) \approx 2$ and $\exp(1.66 - 0.97) \approx 2$.

[15] Appendix I reports some of the firms with the highest and lowest R&D returns in my sample.

Finding large dispersion in investment returns echoes the literature on physical investment. For example, Hsieh and Klenow (2009) document large differences in the return on capital across US firms, whereas the frictionless investment model predicts none. Importantly, this literature argues that the empirical dispersion in the return on capital implies large losses in aggregate production vis-à-vis a return equalizing allocation. Comparing dispersion in both returns in row 1 of Table 1, I find that R&D return dispersion is $1.09/0.77 - 1 \approx 42\%$ larger in my sample suggesting that R&D return dispersion is large.

Table 1: Return Dispersion Across Comparison Groups

| Within Cell | R&D Return | Return on Capital | |
|---|---|---|---|
| | SD | SD | Δ% |
| *A. Across Industries* | | | |
| — | 1.09 | 0.77 | 42% |
| Year | 1.05 | 0.74 | 43% |
| **NAICS3 × Year** | 0.93 | 0.64 | 46% |
| NAICS6 × Year | 0.85 | 0.58 | 45% |
| *B. Across Time* | | | |
| **1975 – 2014** | 0.93 | 0.64 | 45% |
| 1975 – 1990 | 0.74 | 0.46 | 62% |
| 2000 – 2014 | 0.98 | 0.73 | 33% |
| *C. Across Measures* | | | |
| **Patent valuations** | 0.93 | 0.64 | 46% |
| Δ Revenue | 1.11 | 0.64 | 73% |
| Δ Employment | 1.35 | 0.64 | 111% |
| Δ Labor productivity | 1.59 | 0.64 | 150% |

*Note:* Return measures residualized with respect to fixed effects indicated in first column. Column headers SD report standard deviations of return measure. Columns headers Δ% indicate percent difference of Return on R&D dispersion with respect to return in consideration. Returns are measured in logs.

Large dispersion in R&D returns is not surprising if we expect significant measurement error therein. At least three concerns might lead us to that believe. First, we might be concerned about the use of patents to measure innovation as it has been long recognized that not all inventions are patented (Cohen et al., 2000). R&D returns might then reflect whether firms patent their inventions rather than their quantity and value. One important dimension for this consideration is industry differences in patenting conventions that could contribute to R&D return dispersion. For example, patents are considered quite important in life science, but less so in manufacturing (Mezzanotti and Simcoe, 2023). Empirically, I find that the contribution of such cross-industry differences is small to overall R&D return dispersion. Variation across firms in the same 6-digit industry and year accounts for $0.85/1.09 \approx 78\%$ of the overall R&D return dispersion as reported in Panel A of Table 1. It is also not the case that the importance of patenting is a strong predictor for within-industry dispersion. For example, R&D returns are similarly dispersed within life science and manufacturing, as reported in Appendix Table C.2, even though both industries differ significantly in the degree to which they consider patents essential to their intellectual property protection strategy. Furthermore, R&D return dispersion is a robust finding across alternative measures of R&D output that do not rely on patents. For example, the dispersion is $1.11/0.93 - 1 \approx 19\%$ larger when using revenue growth instead of patent valuations to measure R&D output as reported in Panel C. Needless to say, these measures have measurement issues of their own, however, I find that they are highly correlated with my preferred measure and, thus, appear to capture a common factor, as reported in Panel B of Table 2.[16]

A second potential concern relates to the use of estimated patent valuations. Kogan et al. (2017) estimate these by using a non-linear transformation of stock-market return around the patent announcement window. This procedure likely entails some measurement error as it becomes impossible to disentangle other events impacting the firm concurrently from the patent value alone. To the degree that these confounding events are quantitatively important and independent of each other over time, we might thus expect that R&D returns are partly driven by classical measurement error and, thus, uncorrelated over time. Instead, I find that R&D returns are highly persistent over time. The estimate reported in column (1) of Panel in Table 2 implies an annual autocorrelation coefficient of $0.697^{1/5} \approx 0.93$.[17] Importantly, all measures of R&D returns are highly persistent over time as reported in Panel A of Table 2. I propose a structural variance decomposition of R&D returns based on this insight in Appendix D and find that purely transitory innovations in R&D returns, such as classical measurement error, contribute almost none of the overall variation therein. Another concern might be that the measurement approach assigns positive values to all patents by construction, however, there might be a large fraction of patents that are truly worthless in practice (Jaffe and Lerner, 2007). I show in Appendix C.2 that excluding low value patents from R&D returns does not reduce their dispersion. Lastly, Kogan et al. (2017) assume that all patents have an equal likelihood of being granted ex-ante to back out the

[16] An exception to this finding is the measure using changes in labor productivity, where the correlation is weaker. However, this measure is also most dispersed and, thus, might be less reliable.

[17] I estimate the coefficient with 5-year lags to safeguard against mechanical correlation.

value of innovation from the stock return, which only reflects the unexpected component. In practice, this likelihood likely differs across technology classes and might be larger for high value patents. I investigate both concerns in Appendix C.2 and find that adjusting for technology-class specific grant rates does not reduce R&D return dispersion and that allowing the patent grant probability to increase with its valuations increases, rather than decreases, measured R&D return dispersion.

Table 2: R&D Return Consistency across Time and Measures

| Variable | Estimate | Std. err. | $R^2$ | Observations |
|---|---|---|---|---|
| *A. 5-Year Autocorrelation* | | | | |
| Patent valuations | 0.697*** | (0.020) | 45.7% | 7,623 |
| Δ Sales | 0.564*** | (0.024) | 29.6% | 7,455 |
| Δ Employment | 0.552*** | (0.026) | 27.8% | 6,447 |
| Δ Labor Productivity | 0.740*** | (0.026) | 59.6% | 7,411 |
| Δ Market valuation | 0.326*** | (0.029) | 13.8% | 3,731 |
| *B. Correlation with Baseline R&D Returns* | | | | |
| Δ Sales | 0.597*** | (0.033) | 25.1% | 11,688 |
| Δ Employment | 0.551*** | (0.038) | 14.1% | 10,870 |
| Δ Labor Productivity | 0.092* | (0.051) | 0.3% | 11,582 |
| Δ Market valuation | 0.818*** | (0.023) | 47.4% | 6,749 |

*Note:* Each row reports the regression coefficient of a separate regression with dependent and independent variable in logs. Panel A reports 5-year autocorrelation coefficients for alternative measures of R&D returns. The respective R&D return is calculated as the ratio of the variable indicated in column 1 and R&D expenditure at the 5-year level. Variables starting with Δ cumulte non-negative changes in the indicates variable over a 5-year period. Panel B reports contemporaneous correlations with alternative measures of R&D returns as dependent variables and the primary measure of R&D returns as the independent variable. The primary measure is the ratio of patent valuations to R&D expenditure at the 5-year horizon. Alternative measures are calculated as in Panel A. All regressions control for NAICS3× Year fixed effects and standard errors are clustered at the NAICS6 level. See text and Appendix for additional data details.

A third and final concern might be the importance of measurement error related to the distinctions between expected returns, which should be equalized in the frictionless model, and realized returns, which might not be. However, the strong persistence in R&D returns suggests that their dispersion is unlikely due to the gap between expectations and realization as such differences are uncorrelated over time under rational expectations. Nonetheless, I investigate whether "superstar patents", which we might expect to important given the fat-tailed distribution of innovation outcomes documented in (Akcigit and Kerr, 2018), contribute to R&D return dispersion by creating to large differences between expectation and realization. My findings, as reported in Appendix C.2, suggest otherwise. R&D return dispersion is essentially unaffected by winsorizing patent valuations at the 95th percentile.

In summary, my robustness exercises suggest that neither measurement error arising from the use of patents and patent valuations nor classical measurement error appear to be significant drivers of R&D return dispersion.[18] Appendix Table C.1 further investigates robustness with respect to the specification and find that neither expending the aggregation window for R&D inputs and outputs nor changing the timing gap between both leads to lower R&D return dispersion. Focusing on observations with significantly more patents reduces measured dispersion marginally. Thus, I investigate potential economic drivers next.

## 4.2 Economic Drivers of R&D Returns

I investigate potential economic drivers of R&D return dispersion in Table 3 and elaborate on their theoretical foundation in Appendix G. First, I investigate the importance of investment frictions in Panel A. Following the idea that firm-level frictions might distort investment across multiple margins, I investigate whether traditional measures of capital investment frictions correlate with R&D returns. Rows 1 and 2 show mixed results. On the one hand, R&D returns are uncorrelated with the return on capital, which is often considered a summary measure for investment frictions (David et al., 2016). On the other hand, R&D returns are highly correlated with Tobin's $Q$, which is an established measure of investment frictions (Whited and Wu, 2006; Gutiérrez and Philippon, 2017). I also find mixed results for measures of financial frictions. For example, firms with more liquidity, which might be less constrained by cash flow concerns, tend to have lower returns, in line with the idea that they are relatively less constrained. On the other hand, firms with large dividend payments, which presumably are not very constrained either, have larger rather than smaller R&D returns. Finding inconclusive results for measures of financial frictions is surprising as a growing literature argues that intangible capital investments, including R&D, are particularly constrained by them (Brown et al., 2009; Peters and Taylor, 2017).

Second, R&D return dispersion could be driven by firm-specific risk-premia due to differential exposure to aggregate risk. David et al. (2022) argue that risk-premia can explain a significant share of return on capital dispersion. In the case of R&D returns I find no evidence that firms' stock market $\beta$, a common measure of systematic risk, has significant explanatory power, as reported in Panel B. However, it appears that firms whose patent valuations are particularly volatile tend to have higher R&D returns. Such a risk-premium

[18] Appendix C.2 also considers misspecification arising form the presence of fixed costs, acquisitions of innovative firms, and knowledge capital. The robustness exercises reported therein suggests that they are not significant drivers of R&D return dispersion.

could arise if firms' decision makers are not able to fully diversify away the innovation risk.

Table 3: Correlations with R&D Returns

| Variable | Estimate | Std. err. | $R^2$ | Observations |
|---|---|---|---|---|
| *A. Frictions* | | | | |
| Return on Capital | 0.043 | (0.068) | 0.1% | 11,844 |
| Tobin's Q | 0.202*** | (0.030) | 6.3% | 10,471 |
| Liquidity | -0.048** | (0.022) | 0.3% | 10,568 |
| Net-leverage | 0.790 | (0.543) | 0.2% | 11,499 |
| Dividend rate | 36.499*** | (7.142) | 1.5% | 11,499 |
| Knowledge-intensity | -0.494*** | (0.049) | 12.0% | 11,845 |
| Firm age | 0.001 | (0.003) | 0.0% | 11,845 |
| Employment | 0.094*** | (0.019) | 2.5% | 11,813 |
| *B. Risk* | | | | |
| CAPM $\beta$ | 0.001 | (0.065) | 0.0% | 6,799 |
| Valuation risk | 0.493*** | (0.132) | 1.4% | 10,961 |
| *C. Taxation* | | | | |
| R&D user cost $1-\tau$ | -0.527 | (0.617) | 0.1% | 11,247 |
| Alt. R&D user cost $1-\tau$ | -0.160 | (0.572) | 0.0% | 11,517 |
| Public patent involvement | 1.392 | (1.240) | 0.2% | 11,845 |
| Investment tax credits | -0.047*** | (0.017) | 0.2% | 11,208 |
| *D. Inventors* | | | | |
| Inventors | 0.228*** | (0.032) | 7.2% | 11,845 |
| Firm dominance | 0.142*** | (0.045) | 1.4% | 10,477 |
| Inventor specialization | 0.233*** | (0.083) | 0.4% | 11,828 |
| Inventor productivity | 0.300*** | (0.054) | 3.8% | 11,845 |
| *E. Dynamics* | | | | |
| Long-term R&D growth | 0.424*** | (0.054) | 10.2% | 6,525 |
| Long-term TFP growth | 0.451*** | (0.050) | 3.7% | 5,421 |
| Prior excess stock return | 0.260*** | (0.031) | 1.1% | 10,087 |
| Prior TFP growth | 0.316*** | (0.041) | 1.1% | 7,277 |

*Note:* Each row reports the regression coefficient of a separate regression with dependent variable log R&D returns, calculated as the ratio of total patent valuations to R&D expenditure. All regressions control for NAICS3× Year fixed effects and standard errors are clustered at the NAICS6 level. Return on capital is log of the ratio of sales to last periods capital stock. Tobin's Q is the ratio of market valuation to book value in logs. Liquidity and dividend rate are cash and dividends over assets, respectively. Knowledge intensity is the log of theratio of knowledge capital from Ewens et al. (2022) to the sum of physical and knowledge capital. R&D user cost are from Lucking (2019) and mapped from the state-level to the firm either via the patent location or headquarters. Investment tax credits is the lof of the ratio of investment tax to investment. Firm dominance is the average share of inventors employed across firms' technology classes. See Appendix B for details.

Third, I investigate the importance of R&D subsidies in Panel C. As suggested in Hsieh and Klenow (2009), investment subsidies distort returns by reducing the true investment costs vis-à-vis reported costs such that firms with large subsidies earn low reported returns. Using data on state-level R&D tax credits from Lucking (2019), I find that the induced variation in R&D user costs only weakly correlates with R&D returns. Similarly, measure of public co-ownership of firms' patents or overall investment tax credit do not explain a significant share of the overall reported R&D return dispersion. Note, however, that these results only speak to the net-effect of R&D subsidies. For example, I would not detect any correlation with R&D returns if R&D subsidies perfectly offset other investment frictions.

Fourth, a growing literature argues that monopsony power is pervasive in the labor market and quantitatively important for the allocation of workers in the production sector (Card et al., 2018; Lamadon et al., 2022).[19] The literature also finds that high-skilled workers, a group likely including many inventors and research scientists, are more affected by monopsony power and that larger firm tend to have more thereof (Prager and Schmitt, 2021; Seegmiller, 2023; Berger et al., 2022; Yeh et al., 2022).[20] Monopsony power over inventors or other R&D inputs could drive R&D return dispersion due to a markdown term. Firms with more monopsony power restrict their hiring more aggressively to keep wage low and, as a result, create more value per unit of cost. In line with this idea, I find that firms hiring more inventors have larger R&D returns in Panel D.[21] I estimate the contribution of size-dependent monopsony power over inventors to R&D returns structurally in Appendix F and estimate that it can account for around 38% of the overall dispersion and provides a good fit for the high persistence in R&D returns. In addition, I find that firms dominating their inventor labor market, as well as those with more productive or specialized inventors tend to have larger R&D returns as reported in Panel D of Table 3. These findings speak to the potential mechanisms driving monopsony power such as firm-specific human capital or limited outside options (Acemoglu, 1997; Schubert et al., 2023).

Lastly, I investigate the relationship of R&D returns to dynamic variables capturing changing

[19] See also Manning (2011); Friedrich et al. (2021); Kroft et al. (2021); Manning (2021); Sokolova and Sorensen (2021)

[20] There is mounting evidence that especially large tech firms are aware of their market power and attempt to exploit it. For example, it is well known that large Tech firms had agreements between each other not to poach employees in order to keep wages low. Apple, Adobe, Intel, and Google got fined by the Department of Justice in 2010 for illegal non-poaching agreements to keep salaries for tech workers low with further subsequent investigations. See here, here, here. Microsoft only recently announced that it will not enforce its non-compete clauses for employees and was previously sued for their non-poaching agreements. Similar cases have emerged in other industries.

[21] Alternatively, one could interpret this finding as a sign of a size-dependent scale elasticity in the R&D production function. To rationalize the finding, one would need to assume that up-scaling is costlier for already large companies.

firm fortunes in Panel E. For example, R&D return dispersion arises naturally in a context with adjustment costs to R&D investments. Asker et al. (2014) highlight this intuition in the context of capital investment and conclude that adjustment costs contribute to dispersion in the return to capital. In a model with adjustment costs, a positive shock to R&D productivity will lead to a temporary rise in R&D returns as R&D output, which captures R&D inputs and productivity, adjusts faster than R&D inputs alone. In such a world, R&D returns should thus be correlated with R&D growth. In line with this prediction, I find that long-term growth in R&D, i.e., the growth rate thereof from t-1 to t+6, is strongly correlated with R&D returns and can account for around 10% of their dispersion.[22] Similarly, I find a strong correlation with long-term TFP growth, which can be rationalized by the same mechanism. Finally, I also find that prior TFP growth and stock returns are predictive for R&D returns, suggesting, again, that they are associated with firm expansion and other positive events for the firms. Note, however, that these correlation could also be rationalized by a model with size dependent monopsony power as in Berger et al. (2022). I thus estimate the contribution of fixed and quadratic adjustment costs structurally in Appendix Section E. My results suggests that adjustment costs can account for 16% of the dispersion in R&D returns, however, they fail to capture their persistence in the data.

Overall, the explanatory power of the mechanisms considered here appears low, echoing similar results in the return on capital literature (David et al., 2016; David and Venkateswaran, 2019). This finding makes the interpretation of measured R&D wedges difficult, since we do not fully know their source. Nonetheless, the documented dispersion marks a stark deviation from the predictions of a frictionless model and can be interpreted using the formulae developed in Section 2.

## 4.3 R&D Wedges and Impact-Value Factors in Relation

Finally, I investigate the link between R&D wedges and the impact-value factor in Table 4 and find mixed results. On the on hand, markup-based measure suggest slightly positive correlation. For example, I find a significant positive correlation of the R&D return with the profit-implied markup, i.e., revenue divided by revenue minus cost, implying that firms with higher R&D returns also tend to have larger markups. The same relationship, although smaller in absolute magnitude, holds when using the markup measure developed in Loecker et al. (2020). To the degree that markup differences are primarily driven by persistent

[22] I use long-term growth rate to avoid mechanical correlation.

differences in the quality of innovation, as in a model with limit pricing and heterogeneous innovation quality, these results suggest that the impact-value factor might amplify the misallocation due to R&D return dispersion.

On the other hand, patent-based measures suggest a negative correlation. For example, when measuring the impact-value factor as citations over valuations, I find a strong negative correlation. However, this might be mechanical as valuations are used in both measures. The negative correlation is less pronounced when using sales growth instead of patent valuations. I also find a robust negative correlation when using citations over valuations to measure the impact value factor and sales growth over R&D expenditure to measure the R&D wedge. Finally, using the text-based patent quality measure developed in Kelly et al. (2021) as a proxy for the growth impact, I find a strong negative correlation with the R&D return. Thus, if these patent-based measures provide a good proxy for the impact-value factor, then they might partly offset misallocation due to R&D wedges.

In summary, the data does not support strong conclusions on the relationship between R&D wedges and impact-value factors due to conflicting findings. I also investigate the evolution of correlations over time and, again, find mixed results as reported in Appendix Table C.3. While R&D returns tend to be more positively correlated with markup-based measures in the later sample, they are also more negatively correlated with patent-based measures. I provide additional stylized facts on impact-value factors in Online Appendix H.

Table 4: The Relationship of R&D Wedges and Impact-Value Factors

| Impact-Value Factor | Estimate | Standard Error | $R^2$ | Observations |
|---|---|---|---|---|
| *A. Markup-based Measures* | | | | |
| Estimated Markup | 0.030*** | (0.007) | 2.7% | 10,615 |
| Profit-implied Markup | 0.066*** | (0.016) | 4.6% | 11,845 |
| *B. Patent-based Measures* | | | | |
| Citations/Valuations | -0.490*** | (0.034) | 16.4% | 11,845 |
| **Citations/Δ Sales** | -0.077 | (0.054) | 0.3% | 11,688 |
| Citations/Valuations* | -0.201*** | (0.032) | 3.8% | 11,688 |
| Text-Impact/Δ Sales | -0.184*** | (0.053) | 1.9% | 7,481 |

*Note:* Each coefficient stems from a separate regression with the R&D wedge as the independent variable and a measure of the impact-value factors as the dependent variable. The R&D wedge is measured as the ratio of patent valuations over R&D expenditure excepts for the third row, where it is measured as changes in sales over R&D expenditure. Profit-implied markups are measured as one over one minus the profit rate, which is the ratio of profits to sales. Measured markups refer to the $\mu_2$ markup measure from Loecker et al. (2020). Variables are aggregated to the 5-year level as described in the text and the regression specification is in logs. Standard errors are in round brackets and the observation number in rectuangular brackets. All regressions control for NAICS3× Year fixed effects and standard errors are clustered at the NAICS6 level.

# 5 Growth, Wedges, and Policy

## 5.1 Combining Data and Model

Having explored R&D wedges at the micro-level, we can now turn to their macroeconomic implications. I next describe how I combine model and data to estimate the impact of R&D wedges on R&D productivity, growth, and welfare.

**Measurement.** Following Proposition 2, I calculate annual estimates for the Impact of R&D wedges as

$$\hat{\Xi}_t = \frac{\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{\omega}_{it} \cdot (\widehat{1+\Delta_{it}})^{-\frac{\gamma}{1-\gamma}\cdot\hat{\hat{\beta}}_t}}{\left(\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{\omega}_{it} \cdot (\widehat{1+\Delta_{it}})^{-\frac{\gamma}{1-\gamma}\cdot\hat{\hat{\beta}}_t}\right)^{\gamma}} \quad \text{with} \quad \hat{\omega}_{it} = \frac{\hat{\theta}_{it}^{\frac{1}{1-\gamma}}}{\frac{1}{N_t}\sum_{i=1}^{N_t} \hat{\theta}_{it}^{\frac{1}{1-\gamma}}}. \tag{17}$$

Importantly, this approach implicitly assumes that my sample is representative for the US R&D sector. The estimates are biased towards a less negative Impact of R&D wedges if large and established firms tend to be less impacted by frictions and constraints.

I consider two scenarios for adjustment factor $\hat{\hat{\beta}}_t$. In the first case, I assume that R&D wedges and impact-value factors are independent and, thus, set $\hat{\hat{\beta}}_t = 1$. In the second case, I estimate its value based on the estimated coefficient when regressing R&D returns on a proxy for the impact-value factor as in Table 4, but using a centered rolling 10-year window. The adjustment factor is $\hat{\hat{\beta}}_t = \sqrt{1 + 2 \cdot \hat{\beta}_t}$, where $\hat{\beta}_t$ is the regression coefficient. In my preferred specification, I measure the impact-value factor as citations divided by sales growth (fourth row) and report robustness below. I present annual estimates in Appendix Figure C.4.

To get a sense of longer-run developments, I collapse annual estimates using geometric averages. I consider the average over the full sample from 1975 to 2014 as well as the early and late periods, 1975–90 and 2000–14, respectively. Comparing the early and late period gives us a window into long-run changes of the Impact of R&D wedges and their potential impact on economic growth.

Finally, to get an idea of the variability in the estimates, I calculate standard errors using a bootstrapping procedure. For each year, I re-sample firm observations with replacement until I reach the true sample size and calculate the annual aggregates based on the new sample. I repeat this exercise for 1000 bootstrap samples and report the standard deviation of the resulting estimates together with non-parametric 95% confidence intervals.

**Counterfactuals.** Equation 2 allows us to estimate the short-run impact of R&D wedges by comparing the growth rate under the measured impact $\hat{\Xi}_t$ to its hypothetical value under $\Xi_t = 1$. This counterfactual implicitly assumes that offsetting R&D wedges is technologically feasible, which is straight-forward in the case of frictions that can be overcome by targeted subsidies, such as financial frictions, market power over R&D inputs, or R&D subsidies. However, this counterfactual may not be feasible in the case of, e.g., adjustment costs. Nonetheless, the measured Impact of R&D wedges $\hat{\Xi}_t$ is still be informative about whether changes in the economic growth rate arise from R&D wedges or the frictionless growth rate.

I consider two scenarios when estimating the long-run impact on economic growth and welfare. In the first scenario, I set $L_t = L$ and $\phi = 0$, such that a policy setting $\Xi_t = 1$ immediately achieves the frictionless growth rate $g^C$, which I calibrate as $g^C = 1.5\% \cdot \Xi^{-1}$ to match the long-run US growth rate. I refer to this scenario as the endogenous growth case as it achieves a constant long-run growth rate depending on the allocation of R&D resources.

In the second scenario, which I refer to as the semi-endogenous growth case following Jones (1995), I assume that the frictionless growth rate and population dynamics take the form

$$g_t^C = A_t^{-\phi} \cdot L_t^{\gamma} \cdot g^C \qquad \text{with} \quad \phi > 0 \quad \text{and} \quad L_{t+1} = (1+n) \cdot L_t. \tag{18}$$

The parameter $\phi > 0$ determines the degree to which "ideas are getting harder to find" over time, which is key to achieving constant productivity growth in the long-run with a growing population (Bloom et al., 2020). The long-run growth rate in this economy is pinned down by $g = (1+n)^{\gamma/\phi} - 1$, however, changes in the economic environment can have temporary effects on the growth rate and induce permanent changes in the productivity level.[23] In the counterfactual, I assume that the economy is on its long-run growth path before the policy change and trace-out subsequent changes in productivity and consumption. I set population growth to $n = 1\%$ and calibrate $\phi$ to achieve a long-run growth rate of 1.5%.

We are thus in a position to estimate the impact of R&D wedges on the allocation of R&D resources, economic growth and welfare.

[23] To achieve constant growth in the long-run absent of changes in $\{\tilde{g}_t, \Lambda_t, \Xi_t\}$, $A_t^{\phi} \cdot L_t^{\gamma}$ need to be constant as well, such that we can solve for $g$ conditional on population growth rate $n$. We can also solve for the level of technology at any point in time on this growth path, which is given by

$$A_t = \left( \frac{L_t^{\gamma} \cdot \tilde{g} \cdot \Lambda \cdot \Xi}{(1+n)^{\gamma/\phi} - 1} \right)^{1/\phi}.$$

## 5.2 The Long-run View

As discussed in Section 2, R&D efficiency crucially depends on the relationship of R&D wedges and impact-value factors. I first consider the case where both are unrelated, e.g., because of a common impact-value factor. The blue line in Figure 2 plots the annual estimates of $\Xi_t$, while long-run values are reported in Panel A of Table 5. The table also reports bootstrapped standard errors as well as the welfare cost in consumption-equivalent terms.[24]

Table 5: The Impact of R&D Wedges on Economic Growth and Welfare

| Time Horizon | Growth Impact $\Xi - 1$ | | | Welfare Cost | |
|---|---|---|---|---|---|
| | Estimate | Std. Err. | 95% CI | Endogenous | Semi-End. |
| *A. Baseline* | | | | | |
| **1975–2014** | -21.3% | (0.41%) | [-21.9% -20.5%] | 12.3% | 11.7% |
| 1975–1990 | -14.7% | (0.49%) | [-15.4% -13.8%] | 7.6% | 7.4% |
| 2000–2014 | -24.0% | (0.68%) | [-25.0% -22.8%] | 14.5% | 13.7% |
| Δ Change | -10.93% | | | 5.4% | 5.3% |
| *B. Adjusted* | | | | | |
| **1975–2014** | -17.9% | (0.36%) | [-18.4% -17.3%] | 9.8% | 9.4% |
| 1975–1990 | -12.0% | (0.40%) | [-12.6% -11.2%] | 6.0% | 5.9% |
| 2000–2014 | -21.7% | (0.62%) | [-22.6% -20.6%] | 12.6% | 12.0% |
| Δ Change | -11.02% | | | 5.5% | 5.3% |

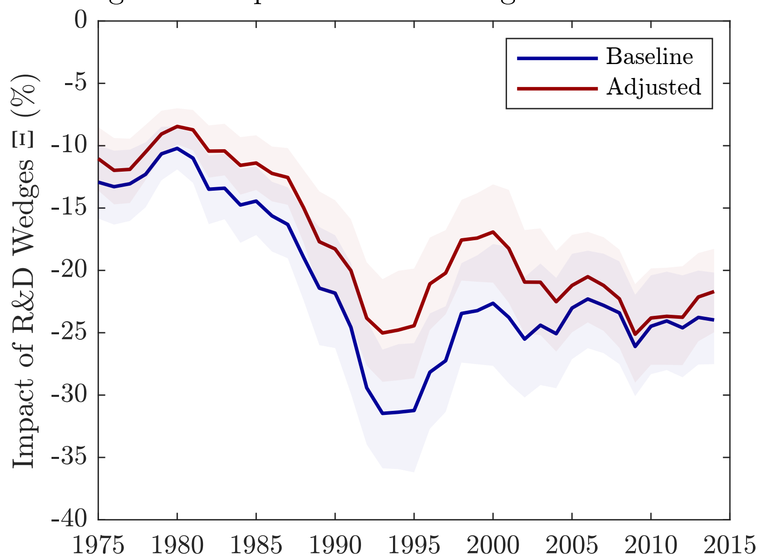
*Notes:* Table reports estimates for impact of R&D wedges across samples together with their implications for welfare. Changes in welfare are in consumption equivalent terms. Standard errors and confidence intervals are calculated using a bootstrapping procedure. See text and Appendix for details.

Frictions, as captured by R&D wedges, have a significantly negative impact on economic growth. I estimate an average growth impact of -21.3% for the entire sample from 1975 to 2014, which suggests a growth rate of 1.9% in absence of R&D wedges based on a realized annual productivity growth rate of 1.5%. Unsurprisingly, such a stark slowdown of economic growth has large welfare consequences. The model suggest that welfare would be 12% higher in absence of R&D wedges. For comparison, Berger et al. (2022) estimate that monopsony in the production sector reduces US output by 21% and welfare by 8%, while Hsieh and Klenow (2009) estimate 30%–40% larger US output in absence of production factor misallocation.[25]

[24] Suppose we start with path $\{Y_t\}$ and want to evaluate alternative $\{Y_t'\}$. Then, the consumption-equivalent welfare improvement $\Delta_C$ is the permanent proportional change in the old consumption path, such that the household is equally well-off under both new paths: $\mathcal{W}(\{Y_t \cdot (1 + \Delta_C)\}) = \mathcal{W}(\{Y_t'\})$.

[25] Arkolakis et al. (2012) and Lucas (2003) famously estimate welfare cost of autarky and business cycles, respectively, around 1%, however, their methods are less closely connected to this paper.

Figure 2: Impact of R&D Wedges over Time




*Notes:* Figure reports annual estimates for R&D efficiency $\Xi_t$. The baseline estimate assumes that R&D wedges are independently distributed from the impact-value factor as in Corollary 1. Adjusted estimates account for potential correlation following Proposition 2 and estimate the adjustment factor over a 10-year rolling window using citations over sales changes as a proxy for the impact-value factor as in specification 3 in Table 4. The shaded area covers the 90% confidence interval calculated using a bootstrapping procedure redrawing firm observations annually for 1000 samples. See text and Appendix for details.

The annual estimates further suggest that declining Impact of R&D wedges might help to explain the long-run growth slowdown (Syverson, 2017). Figure 2 reveals that the estimated Impact of R&D wedges declines over time with a sharp downturn and partial recover during the 1990s, potentially reflecting dynamics in the Dot-Com boom. Comparing the estimates for the 1975–90 and 2000–14 period in Table 5, I find that the Impact of R&D wedges declined from -15% to -24% with an associated welfare loss around 5%. This decline implies an $\frac{24\%-15\%}{1-15\%} \approx 11\%$ slower growth rate in the short-run, which can account for $\frac{11\%}{35\%} \approx 31\%$ of the overall decline in economic growth documented in Aghion et al. (2023).[26]

Adjusting for the impact-value factor reduces the cost of R&D returns marginally, but leaves their evolution essentially unaffected. The long-run estimated Impact of R&D wedges is -18%, which is slightly better than the unadjusted estimate of -21%. Nonetheless, the estimates welfare cost of 9%–10% remain large. Finally, the change in the economic growth rate implied by the evolution of the Impact of R&D Wedges remains -11%.

[26] Productivity growth for non-farm private industries declined from 1.81% for the 1948–1995 period to 1.18% for the 2005–2018 period, a $\frac{1.81\%-1.18\%}{1.81\%} \approx 35\%$ reduction. See Figure A1 in their paper.

## 5.3 Discussion

Before discussing potential drivers of rising frictions and gap to the growth frontier, I want to discuss a range of potential concerns including measurement.

First, not all inventions are patented and, thus, using patent valuations as a measure of private value created from R&D is necessarily an incomplete measure, even if it might be our most reliable one. I consider alternative measures of R&D output in Panel A of Table 6 as robustness and find that using patent valuations leads to the most conservative estimate for the impact of R&D wedges. Its decline over time is more (less) pronounced when using sales (employment) growth instead of patent valuations.

Second, we do not have a convincing measure for the impact-value factor. My preferred specification estimates adjustment factor $\tilde{\beta}$ by regressing citations over sales growth on R&D returns, but I consider two alternatives in Panel B of Table 6. In the first alternative, I estimate the adjustment factor by regressing citations over valuations on sales growth over R&D expenditure. The associated estimates make R&D wedges slightly less costly and suggest that changes over time only led to a 5% reduction in economic growth. In the second alternative, I use the profit-based measure of the impact-value factor to estimate the adjustment term and find that the resulting estimates are slightly larger, while changes over time continue to hover still imply a growth impact around -11%.

Third, I investigate the impact of entry and exit over time and find similar results for all and continuing firms only.[27] The estimates continue to suggest a significantly negative impact of R&D wedges on growth of -16%, while changes over time among continuing firms can account for a 9% reduction in economic growth.

Fourth, measurement error is an important consideration regardless of the precise measure of R&D wedges, however, I find little evidence for significant measurement error in practice. I consider two sources in detail in Appendix D. First, the outcome of each innovation effort is uncertain and, thus, we might be concerned that some of the variation in measured R&D wedges is due to firms being more or less lucky in their research projects. I attempt to estimate the contribution of this channel in a bootstrapping approach in which I first redraw firms' patent valuations and citations, and then calculate how far aggregated values are from the true expectation as measured by the firms actual patent valuation and citations. Naturally, these differences are smaller for firms with more patents by the law of large

[27]See Appendix C.6 on how I calculate estimates for continuing firms.

numbers. My results suggest that there is a quite limited amount of variation that might be explained by this source of measurement error. Second, firms might be subject to ex-post firm-level shocks that have a uniform effect on the value of their R&D output. Such variation is not accounted for by the bootstrapping approach as it is common across all inventions within a given period and, thus, does not wash out. I propose an estimation methodology for this source of variation using a GMM estimator in Appendix D. The main idea is to exploit the persistence of R&D returns to estimate the contribution of non-persistent variation, such as one-off luck or firm-level measurement error, to the overall dispersion in R&D returns . My results suggest almost no contribution of such "measurement error" for my main estimates as reported in Panel D, however, I do find evidence for significant contribution when using changes in sales to measure the private value created from R&D.

Table 6: R&D Wedges, Economic Growth and Welfare — Robustness

| Specification | Growth Impact $\Xi - 1$ | | | | Welfare Cost of $\Delta$ | |
|---|---|---|---|---|---|---|
| | 1975–2014 | 1975–90 | 2000–14 | $\Delta$ | End. | Semi-End. |
| *A. Value of Innovation* | | | | | | |
| **Patent Valuations** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| $\Delta$ Sales | -25.3% | -19.5% | -31.7% | -15.1% | 7.9% | 7.7% |
| $\Delta$ Employment | -40.1% | -39.1% | -43.1% | -6.6% | 3.1% | 3.0% |
| *B. Impact-Value Adjustment* | | | | | | |
| **Citations/ $\Delta$ Sales** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| Citations/ Valuations | -15.3% | -12.3% | -16.8% | -5.2% | 2.4% | 2.3% |
| Profit-Based | -22.8% | -15.5% | -25.3% | -11.6% | 5.8% | 5.7% |
| *C. Entry & Exit* | | | | | | |
| **All Firms** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| Continuing Firms | -16.1% | -11.3% | -19.1% | -8.8% | 4.2% | 4.2% |
| *D. Measurement Error* | | | | | | |
| **Unadjusted** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| Adjusted | -17.8% | -11.9% | -21.5% | -10.9% | 5.4% | 5.3% |

*Notes:* Table reports estimates for impact of R&D wedges across samples together with their implications for welfare. Changes in welfare are in consumption equivalent terms. See text and Appendix for details.

Finally, I present robustness checks around the specification in Appendix Table C.6 investigating the choice of industry fixed effects, minimum number of patents, and aggregation horizon. My main findings are qualitatively and quantitatively robust to these choices.

## 5.4 Drivers of Rising R&D Wedges

Before concluding, I want to briefly discuss two potential drivers of the documented empirical patterns: Rising market power and declining fiscal support of R&D. While I refrain from attempting to quantify the importance of the individual channels, they still serve as a potential lens through which to view the results in the previous section.

First, A growing literature documents rising concentration in the corporate sector, which might go hand-in-hand with rising market power (Autor et al., 2020; de Ridder, 2023; Aghion et al., 2023). Concentration has also been rising in the R&D sector. I document in Appendix Figure C.1 that the share of patents granted to the top 5% of firms has been rising from 49% in 1975–90 to 67% in 2000–14 with a similar rise in the concentration of inventors and sales, and slightly lower, but still meaningful, increases in the concentration of patent valuations and R&D expenditure. A potential consequence of rising concentration is increasing market power in input markets. In fact, there is some concern that larger firms are increasingly able to exert market power in the labor market, even for high-skilled labor. (Shi, 2023; Seegmiller, 2023; Schubert et al., 2023; Yeh et al., 2022). Such a channel could lead to rising dispersion in R&D wedges capturing heterogeneity in labor market power over R&D workers. Rising concentration in the corporate sector, thus, could explain rising dispersion in R&D returns leading to declining R&D efficiency and, thereby, slower (short-run) economic growth.

Second, there has been a large decline in federal support for corporate research (Arora et al., 2020). As documented in Appendix Figure C.2, the share of private R&D that is financed by the federal government has declined from 40% in 1970 to 10% in 2010. The decline might have contributed to rising dispersion in R&D wedge and, thus, to a decline in R&D efficiency along multiple dimensions. For example, government R&D funds might have been deployed to alleviate firms' need for external finance, which might be subject to financial frictions (Howell, 2017). Similarly, government might have used the funds implicitly to subsidize firms with market power of R&D inputs and, thereby, offset some of the distortions of market power. Alternatively, federal funding and R&D employment might have acted as an outside option for R&D workers, which limited private firms' labor market power in the first place. In either case, it is possible that the stark decline in federal funding of R&D also had led to a reduction in the allocative efficiency in the R&D sector and, thus, to slower growth, at least in the short-run.

# 6 Conclusion

This paper presents evidence suggesting that frictions, and their impact on the allocation of R&D resources, were an important driver of the decline in aggregate R&D productivity and resulting slowdown of economic growth experienced by the US in the previous two decades. I reach this conclusion based on a growth accounting framework allowing for firm heterogeneity in their R&D productivity, frictions captured by R&D wedges, and the degree to which firms are able to benefit from their inventions, the impact-value factor. The model growth rate can be decomposed in the frictionless competitive growth rate and an adjustment factor capturing frictions, the Impact of R&D wedges.

To estimate the latter, I measure the model fundamentals using data on R&D expenditure and patenting activity for a sample of US-listed firms with significant R&D activity in the 1975–2014 period. Within the context of the model, we can measure the R&D wedges using R&D returns, i.e., the ratio of value created from R&D to its costs. I measure R&D returns as the ratio of patent valuations divided by R&D expenditure over a 5-year window and show that there are large and persistence differences therein. In contrast, the frictionless model without subsidies predicts return equalization and associates R&D return dispersion with frictions. Measured R&D return dispersion persists in a large set of robustness exercises and measurement error adjustments. Lastly, I investigate economic drivers of R&D returns and find evidence suggesting adjustment frictions, financial frictions, and monopsony power over inventors as potential drivers, however,most variation in R&D returns remains unexplained.

Next, I estimate the aggregate Impact of R&D wedges by combining model formulae with the firm-level data. My estimates suggests that frictions reduce US economic growth significantly and increasingly so. I estimate for the full sample that economic growth was 18% slower due to frictions, implying a welfare cost of 11% in consumption-equivalent terms. Furthermore, I find that rising frictions can account for an 11% lower growth rate for 2000–14 compared to 1975–90, which can account for about 30% of the observed productivity slowdown. Jointly, the evidence suggests that private frictions matter for economic growth and increasingly so.

These findings suggest important avenues for future research. Most importantly, more research is needed to understand the underlying forces driving rising frictions. A thorough understanding of the variation in R&D wedges and impact-value factors will allow for the development of potentially targeted policies and is thus essential for improving US R&D productivity and economics growth.

# References

**Abrams, David S., Ufuk Akcigit, and Grennan Jillian**, "Patent Value and Citations: Creative Destruction or Strategic Disruption?," *NBER Working Paper 19647*, 2018.

**Acemoglu, Daron**, "Training and Innovation i n an Imperfect Labour Market," 1997.

**_ and Dan Cao**, "Innovation by entrants and incumbents," *Journal of Economic Theory*, 2015, *157*, 255–294.

**_ , Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William R. Kerr**, "Innovation, Reallocation, and Growth," *American Economic Review*, 2018, *108*, 3450–3491.

**Aghion, Philippe and Peter Howitt**, "A Model of Growth Through Creative Destruction," *Econometrica*, 1992, *60*, 323–351.

**_ , Antonin Bergeaud, Timo Boppart, Peter J Klenow, and Huiyu Li**, "Good Rents versus Bad Rents: R&D Misallocation and Growth," 2022.

**_ , _ , _ , Peter J. Klenow, and Huiyu Li**, "A Theory of Falling Growth and Rising Rents," *Review of Economic Studies*, 2023, *forthcoming.*

**_ , Ufuk Akcigit, and Peter Howitt**, *What Do We Learn From Schumpeterian Growth Theory?*, Vol. 2, Elsevier B.V.,

**Akcigit, Ufuk and Sina T. Ates**, "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory," *American Economic Journal: Macroeconomics*, 2021, *13*, 257–298.

**_ and William R. Kerr**, "Growth Through Heterogeneous Innovations," *Journal of Political Economy*, 2018, *126*, 1374–1443.

**_ , Douglas Hanley, and Stefanie Stantcheva**, "Optimal Taxation and R&D Policies," *Econometrica*, 2022, *90*, 645–684.

**Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare**, "New trade models, same old gains?," *American Economic Review*, 2 2012, *102*, 94–130.

**Arora, Ashish, Sharon Belenzon, Andrea Patacconi, and Jungkyu Suh**, *The changing structure of American innovation: Some cautionary remarks for economic growth*, Vol. 20,

**Asker, John, Allan Collard-Wexler, and Jan De Loecker**, "Dynamic Inputs and Resource (Mis)Allocation," *Journal of Political Economy*, 2014, *122*, 1013–1063.

**Autor, David, David Dorn, Christina Patterson, and John Van Reenen**, "The Fall of the Labor Share and the Rise of Superstar Firms," *The Quarterly Journal of Economics*, 2020, *135*, 645–709.

**Ayerst, Stephen**, "Innovator Heterogeneity, R&D Misallocation and the Productivity Growth Slowdown," 2022.

**Baber, William R, Patricia M Fairfield, and James A Haggard**, "The Effect of Concern about Reported Income on Discretionary Spending Decisions: The Case of Research and Development," *The Accounting Review*, 1991, *66*, 818–829.

**Berger, David W., Kyle Herkenhoff, and Simon Mongey**, "Labor Market Power," *American Economic Review*, 2022, *112*, 1147–1193.

**Bloom, Nicholas, Charles I. Jones, John Van Reenen, and Michael Webb**, "Are Ideas Getting Harder to Find?," *American Economic Review*, 2020, *110*, 1104–1144.

**_ , Mark Schankerman, and John Van Reenen**, "Identifying Technology Spillovers and Product Market Rivalry," *Econometrica*, 2013, *81*, 1347–1393.

**Brown, James R., Steven M. Fazzari, and Bruce C. Petersen**, "Financing innovation and growth: Cash flow, external equity, and the 1990s r&d boom," *Journal of Finance*, 2 2009, *64*, 151–185.

**Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline**, "Firms and labor market inequality: Evidence and some theory," *Journal of Labor Economics*, 2018, *36*, S13–S70.

**Chen, Zhao, Zhikuo Liu, Juan Carlos Suárez Serrato, and Daniel Yi Xu**, "Notching R&D investment with corporate income tax cuts in China," *American Economic Review*, 2021, *111*, 2065–2100.

**Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber**, "Does Indivisible Labor Explain the Difference between Micro and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities," *NBER Macroeconomics Annual*, 2012, *27*, 1–56.

**Cohen, Wesley M, Richard R Nelson, and John P Walsh**, "Protecting their intellectual assets: Appropriability conditions and why U.S. manufacturing firms patent (or not)," *NBER Working Paper 7552*, 2000.

**Crouzet, Nicolas and Janice C Eberly**, "Rents and Intangible Capital: a Q+ Framework," *The Journal of Finance*, 2023, *78*, 1873–1916.

**David, Joel M.**, "The Aggregate Implications of Mergers and Acquisitions," *The Review of Economic Studies*, 2021, *88*, 1796–1830.

_ **and Venky Venkateswaran**, "The sources of capital misallocation," *American Economic Review*, 2019, *109*, 2531–2567.

_ **, Hugo A. Hopenhayn, and Venky Venkateswaran**, "Information, Misallocation, and Aggregate Productivity," *The Quarterly Journal of Economics*, 5 2016, *131*, 943–1005.

_ **, Lukas Schmid, and David Zeke**, "Risk-adjusted capital allocation and misallocation," *Journal of Financial Economics*, 2022, *145*, 684–705.

**de Ridder, Maarten**, "Market Power and Innovation in the Intangible Economy," *American Economic Review*, 2023, *forthcoming.*

**Dukes, Roland E, Thomas R Dyckman, and John A Elliott**, "Accounting for Research and Development Costs: The Impact on Research and Development Expenditures," *Journal of Accounting Research*, 1980, *18*, 1–26.

**Ewens, Michael, Ryan Peters, and Sean Wang**, "Measuring Intangible Capital with Market Prices," 2022.

**Fons-Rosen, Christian, Pau Roldan-Blanco, and Tom Schmitz**, "The Effects of Startup Acquisitions on Innovation and Economic Growth," 2023.

**Friedrich, Benjamin, Lisa Laun, Costas Meghir, and Luigi Pistaferri**, "Earnings Dynamics and Firm-Level Shocks," *NBER Working Paper Series*, 2021, *25786.*

**Gancia, Gino and Fabrizio Zilibotti**, "Horizontal Innovation in the Theory of Growth and Development," *Handbook of Economic Growth*, 2005, *1*, 111–170.

**Goolsbee, Austan**, "Investment subsidies and wages in capital goods industries: To the workers go the spoils?," *National Tax Journal*, 2003, *56*, 153–165.

**Gutiérrez, Germán and Thomas Philippon**, "Declining Competition and Investment in the US," 2017.

**Han, Chirok and Peter C.B. Phillips**, "GMM estimation for dynamic panels with fixed effects and strong instruments at unity," *Econometric Theory*, 2010, *26*, 119–151.

**Howell, Sabrina T.**, "Financing innovation: Evidence from R&D grants," *American Economic Review*, 2017, *107*, 1136–1164.

**Hsieh, Chang-Tai and Peter J. Klenow**, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 2009, *124*, 1403–1448.

**Jaffe, Adam B. and Josh Lerner**, *Innovation and Its Discontents: How Our Broken Patent System is Endangering Innovation and Progress, and What to Do About It*, Princeton University Press, 2007.

**Jones, Charles I.**, "Time Series Tests of Endogenous Growth Models," *The Quarterly Journal of Economics*, 1995, *110*, 495–525.

**Kelly, Bryan, Dimitris Papanikolaou, Amit Seru, and Matt Taddy**, "Measuring Technological Innovation over the Long Run," *American Economic Review: Insights*, 9 2021, *3*, 303–20.

**Klette, Tor Jakob and Samuel Kortum**, "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 2004, *112*, 986–1018.

**Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman**, "Technological Innovation, Resource Allocation, and Growth," *The Quarterly Journal of Economics*, 5 2017, *132*, 665–712.

**Kroft, Kory, Yao Luo, Magne Mogstad, and Bradley Setzler**, "Imperfect Competition and Rents in Labor and Product Markets: The Case of the Construction Industry *," 2021.

**König, Michael, Kjetil Storesletten, Zheng Song, and Fabrizio Zilibotti**, "From Imitation to Innovation: Where Is All That Chinese R&D Going?," *Econometrica*, 2022, *90*, 1615–1654.

**Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler**, "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market," *American Economic Review*, 1 2022, *112*, 169–212.

**Lev, Baruch, Bharat Sarath, and Theodore Sougiannis**, "R&D Reporting Biases and Their Consequences," *Contemporary Accounting Research*, 2005, *22*, 977–1026.

**Loecker, Jan De, Jan Eeckhout, and Gabriel Unger**, "The Rise of Market Power and the Macroeconomic Implications," *The Quarterly Journal of Economics*, 2020, *135*, 561–644.

**Lucas, Robert E.**, "Macroeconomic priorities," *American Economic Review*, 3 2003, *93*, 1–14.

**Lucking, Brian**, "Do R&D Tax Credits Create Jobs?," 2019.

**Manera, Andrea**, "Competing for Inventors: Market Concentration and the Misallocation of Innovative Talent *," 2022.

**Manning, Alan**, *Monopsony in Motion: Imperfect Competition in Labor Markets*, Princeton University Press, 2003.

_ , *Imperfect competition in the labor market*, Vol. 4, Elsevier B.V., 2011.

_ , "Monopsony in Labor Markets: A Review," *ILR Review*, 1 2021, *74*, 3–26.

**Mezzanotti, Filippo**, "Roadblock to Innovation: The Role of Patent Litigation in Corporate R&D," *Management Science*, 2021.

_ **and Timothy Simcoe**, "Innovation and Appropriability: Revisiting the Role of Intellectual Property," *NBER Working Paper No. 31428*, 2023.

**Midrigan, Virgiliu and Daniel Yi Xu**, "Finance and misallocation: Evidence from plant-level data," *American Economic Review*, 2014, *104*, 422–458.

**Olmstead-Rumsey, Jane**, "Market Concentration and the Productivity Slowdown," 2022.

**Peters, Michael**, "Heterogeneous Markups, Growth, and Endogenous Misallocation," *Econometrica*, 2020, *88*, 2037–2073.

**Peters, Ryan H. and Lucian A. Taylor**, "Intangible capital and the investment-q relation," *Journal of Financial Economics*, 2017, *123*, 251–272.

**Phillips, Gordon M and Alexei Zhdanov**, "R&D and the Incentives from Merger and Acquisition Activity," *The Review of Financial Studies*, 2013, *26*, 34–78.

**Prager, Elena and Matt Schmitt**, "Employer Consolidation and Wages: Evidence from Hospitals," *American Economic Review*, 2 2021, *111*, 397–427.

**Restuccia, Diego and Richard Rogerson**, "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 10 2008, *11*, 707–720.

**Romer, Paul M.**, "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 1986, *94*, 1002–1037.

**Romer, Paul M**, "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98.*

**Schubert, Gregor, Anna Stansbury, and Bledi Taska**, "Employer Concentration and Outside Options," 2023.

**Seegmiller, Bryan**, "Valuing Labor Market Power : The Role of Productivity Advantages," 2023.

**Shi, Liyan**, "Optimal Regulation of Noncompete Contracts," *Econometrica*, 2023, *91*, 425–463.

**Sokolova, Anna and Todd Sorensen**, "Monopsony in Labor Markets: A Meta-Analysis," *ILR Review*, 1 2021, *74*, 27–55.

**Solow, Robert M.**, "A Contribution to the Theory of Economic Growth," *The Quarterly Journal of Economics*, 1956, *70*, 65–94.

**Syverson, Chad**, "Challenges to Mismeasurement Explanations for the US Productivity Slowdown," *Journal of Economic Perspectives*, 5 2017, *31*, 165–186.

**Tambe, Prasanna, Lorin M. Hitt, Daniel Rock, and Erik Brynjolfsson**, "IT, AI and the Growth of Intangible Capital," *SSRN Electronic Journal*, 2019.

**Terry, Stephen J.**, "The Macro Impact of Short-Termism," *Econometrica*, 2023, *91*, 1881–1912.

**_ , Toni M. Whited, and Anastasia A. Zakolyukina**, "Information Versus Investment," *Review of Financial Studies*, 2022, *36*, 1148–1191.

**Whited, Toni M and Guojun Wu**, "Financial Constraints Risk," *The Review of Financial Studies*, 2006, *19*, 531–559.

**Wilson, Daniel J.**, "Beggar Thy Neighbor? The In-State, Out-of-State, and Aggregate Effects of R&D Tax Credits," *The Review of Economics and Statistics*, 2009, *91*, 431–436.

**Yeh, Chen, Claudia Macaluso, and Brad Hershbein**, "Monopsony in the U.S. Labor Market," *American Economic Review*, 2022, *112*, 2099–2138.

# Appendix

## A Model Appendix

### A.1 Proofs

*Proof of Proposition 1.* The proof of the proposition is entirely algebraic. Firstly, defining $\theta_{it} = \varphi_{it} \cdot V_{it}$ we can solve for firms' demand for R&D inputs as

$$\ell_{it} = \left( \frac{\theta_{it} \cdot \gamma}{(1 + \Delta_{it}) \cdot W_t} \right)^{\frac{1}{1-\gamma}}.$$

Plugging into the R&D resource constraint, we can solve for the R&D input price:

$$\frac{W_t}{\gamma} = L_t^{-(1-\gamma)} \cdot \left( \int_0^1 (\theta_{it}/(1 + \Delta_{it}))^{\frac{1}{1-\gamma}} \cdot di \right)^{1-\gamma}.$$

Next, using the firm's first order condition, we can express the economic growth rate as

$$g_t = \int_0^1 \zeta_{it} \cdot \ell_{it} \cdot \frac{W_t}{\gamma} \cdot di.$$

Plugging in the definition of the wage and firms' R&D labor demand, we have

$$g_t = L_t^{\gamma} \cdot \frac{\int_0^1 \zeta_{it} \cdot \theta_{it}^{\frac{1}{1-\gamma}} \cdot (1 + \Delta_{it})^{-\frac{\gamma}{1-\gamma}} \cdot di}{\left( \int_0^1 \theta_{it}^{\frac{1}{1-\gamma}} \cdot (1 + \Delta_{it})^{-\frac{1}{1-\gamma}} \cdot di \right)^{\gamma}}.$$

Some rearrangement yields the formulae in the proposition.

□

*Proof of Corollary 1.* The formula follows immediately since the terms in the nominator and denominator are expected values with normalized R&D productivity $\omega_{it}$ acting as a probability weight. Furthermore, and by Jensen's inequality, $\Xi_t \leq 1$ with equality in absence of dispersion in R&D wedges. The final statement follows immediately from the second order approximation provided in Lemma 1. □

*Proof of Proposition 3.* The planner problem is given by

$$\begin{aligned}\max \qquad g_t &= \int_0^1 \zeta_{it} \cdot z_{it} \cdot V_{it} \cdot di \\ \text{s.t.} \quad L_t = \int_0^1 \ell_{it} \cdot di \quad &\text{and} \quad z_{it} = \varphi_{it} \cdot \ell_{it}^{\gamma}\end{aligned}$$

The first order conditions give rise to R&D input demand

$$\ell_{it} = \left( \frac{\zeta_{it} \cdot \theta_{it} \cdot \gamma}{\lambda_t^W} \right)^{\frac{1}{1-\gamma}}, \tag{A.1}$$

where $\lambda_t^W$ is the shadow wage.

One can confirm immediately, that the implied allocation coincides with the competitive equilibrium iff $\zeta_{it} \cdot (1 + \Delta_{it})$ is a constant. All proportional level differences are absorbed into the shadow wage and, thus, do not affect the allocation across firms.

Thus, the planner can implement the growth maximizing allocation by setting $1 + \Delta_{it} = 1/\zeta_{it}$.

□

**Lemma 1.** *The second-order approximation of $\Xi_t$ around $\zeta_{it} = \zeta$ and $\Delta_{it} = \Delta$ is given by*

$$\Xi_t \approx \exp\left( -\frac{1}{2} \cdot \frac{\gamma}{1-\gamma} \left( \sigma_\Delta^2 + 2 \cdot \sigma_{\Delta,\zeta} \right) \right), \tag{A.2}$$

*where $\sigma_\Delta^2$ is the weighted variance of log R&D wedges and $\sigma_{\Delta,\zeta}$ is the $\omega_{it}$-weighted covariance of log R&D and Impact-Value factors. The approximation is precise if all variables are jointly log-normal and, in this case, weights are unnecessary for calculating the variance and covariance.*

The proof for this Lemma is provided in the Online Appendix as it is lengthy, but standard.

*Proof of Proposition 2.* The proof for proposition follows by noting that the second-order approximation of $\Xi_t$ in Lemma 1 can be expressed as

$$\Xi_t \approx \exp\left( -\frac{1}{2} \frac{\gamma}{1-\gamma} \sigma_\delta^2 \cdot \tilde{\beta} \right) \quad \text{with} \quad \tilde{\beta} = 1 + 2 \cdot \frac{\sigma_{\delta,\zeta}}{\sigma_\delta^2}.$$

In turn, it is straight-forward to show that a second order approximation of the formula in Proposition 2 yields the same expression. □

## A.2 Extensions

**Specialization of R&D inputs.** There is a long tradition in labor economics arguing that workers might not be perfectly substitutable across firms or that firms are not perfect substitutes from the perspective of workers (Card et al., 2018). Such forces can be incorporated in the model by augmenting the R&D resource constraint to

$$L_t = \left( \int_0^1 \ell_{it}^{1+\xi} \cdot di \right)^{\frac{1}{1+\xi}}, \tag{A.3}$$

where $\xi > 0$ captures increasing marginal costs of R&D inputs to a given firm. Resultingly, firms' wages are potentially heterogeneous and take the form $W_{it} = W_t \cdot \ell_{it}^{\xi}$, where $W_t$ is a common factor clearing the labor market. Firms' first-order conditions are given by

$$\left. \frac{\partial z_{it}}{\partial \ell_{it}} \right|_{\ell_{it}=\ell_{it}^*} \cdot V_{it} = (1+\Delta_{it}) \cdot W_t \cdot \ell_{it}^{\xi}. \tag{A.4}$$

Proposition 4 highlights that the main results carry over to this alternative setup, however, the effective scale elasticity is lower. Resultingly, frictions tend to be less costly for larger $\xi$ as reallocation of resources becomes less beneficial in a world with specialized inputs.

**Proposition 4.** *Under equations (2), (A.3), (A.4), and (5), we can express the economic growth rate in a Competitive Growth Equilibrium as the product of three terms:*

$$g_t = \underbrace{\frac{L_t^{\gamma}}{A_t^{\phi}} \cdot \left( \int_0^1 (\theta_{it} \cdot \zeta_{it})^{\frac{1}{1-\tilde{\gamma}}} \, di \right)^{1-\tilde{\gamma}}}_{= \text{ Frontier Growth Rate } g_t^F} \cdot \underbrace{\left( \int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it}^{\frac{1}{1-\tilde{\gamma}}} di \right)^{\tilde{\gamma}-1}}_{\equiv \text{ Policy Opportunity } \Lambda_t} \cdot \underbrace{\frac{\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot (1+\Delta_{it})^{-\frac{\tilde{\gamma}}{1-\tilde{\gamma}}} di}{\left( \int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\tilde{\gamma}}} di \right)^{\tilde{\gamma}}}}_{\equiv \text{ R\&D Efficiency } \Xi_t}, \tag{A.5}$$

*where* $\tilde{\zeta}_{it} = \zeta_{it} / \left( \int_0^1 \omega_{it} \cdot \zeta_{it} di \right)$ *and* $\omega_{it} = \theta_{it}^{\frac{1}{1-\tilde{\gamma}}} / \left( \int_0^1 \theta_{it}^{\frac{1}{1-\tilde{\gamma}}} di \right)$ *are the normalized impact-value factor and an R&D productivity weight, respectively, and* $\tilde{\gamma} \equiv \frac{\gamma}{1+\xi}$ *is the adjusted scale elasticity.*

*Proof.* R&D input demand is given by

$$\ell_{it} = \left( \frac{\theta_{it} \cdot \gamma}{(1+\Delta_{it}) \cdot W_t} \right)^{\frac{1}{1-\gamma+\xi}}.$$

We can then solve for the growth rate using the R&D input demand and supply constraint:

$$g_t = \frac{L_t^\gamma}{A^\phi} \cdot \frac{\int_0^1 \zeta_{it} \cdot \theta_{it}^{\frac{1+\xi}{1-\gamma+\xi}} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma+\xi}} \cdot di}{\left(\int_0^1 \theta_{it}^{\frac{1+\xi}{1-\gamma+\xi}} \cdot (1+\Delta_{it})^{-\frac{1+\xi}{1-\gamma+\xi}} \cdot di\right)^{\frac{\gamma}{1+\xi}}}.$$

Defining $\tilde{\gamma} = \frac{\gamma}{1+\xi}$ and some rearrangement yields the formulae in the proposition.

□

**Multiple R&D lines.** Consider an alternative version of the model with multiple R&D lines per firm. I will index a firm by $i \in \mathcal{I}$ and a R&D line by $j \in \mathcal{J}_i$. The production function is given by

$$z_{ij} = \varphi_{ij} \cdot \ell_{ij}^\gamma. \tag{A.6}$$

Firms' first order conditions for R&D inputs at the R&D line level are

$$\gamma \ell_{ij}^{1-\gamma} \cdot \theta_{ij} = (1+\Delta_{ij})W. \tag{A.7}$$

We can solve for the R&D wage as

$$\frac{W}{\gamma} = L^{-(1-\gamma)} \left( \int_{\mathcal{I}} \Big( \sum_{j \in \mathcal{J}_\rangle} (\theta_{ij}/(1+\Delta_{ij}))^{\frac{1}{1-\gamma}} \Big) di \right)^{1-\gamma}. \tag{A.8}$$

The economic growth rate is then

$$g = \int_0^1 \left( \sum_{j \in \mathcal{J}_i} \zeta_{ij} \cdot z_{ij} \cdot V_{ij} \right) \cdot di = \frac{L^\gamma}{A_t^\phi} \cdot \frac{\int_0^1 \left( \sum_{j \in \mathcal{J}_i} \zeta_{ij} \cdot \theta_{ij}^{\frac{1}{1-\gamma}} \cdot (1+\Delta_{ij})^{-\frac{\gamma}{1-\gamma}} \right) \cdot di}{\left( \int_0^1 \left( \sum_{j \in \mathcal{J}_i} \theta_{ij}^{\frac{1}{1-\gamma}} \cdot (1+\Delta_{ij})^{-\frac{1}{1-\gamma}} \right) \cdot di \right)^\gamma}. \tag{A.9}$$

Next, consider the inputs at the firm level, measured as

$$\begin{aligned} 1+\Delta_i &= \frac{\sum_{j \in \mathcal{J}_i} \theta_{ij} \cdot \ell_{ij}^\gamma}{W \cdot \sum_{j \in \mathcal{J}_i} \ell_{ij}} = \sum_{j \in \mathcal{J}_i} \frac{\ell_{ij}}{\ell_i} \cdot (1+\Delta_{ij}) \\ \zeta_i &= \frac{\sum_{j \in \mathcal{J}_i} z_{ij} \cdot V_{ij}^P}{\sum_{j \in \mathcal{J}_i} z_{ij} \cdot V_{ij}} = \sum_{j \in \mathcal{J}_i} \frac{\theta_{ij} \cdot \ell_{ij}^\gamma}{\sum_{j \in \mathcal{J}_i} \theta_{ij} \cdot \ell_{ij}^\gamma} \cdot \zeta_{ij} \\ \theta_i &= (1+\Delta_i) \cdot \tilde{W}^\gamma \cdot (\tilde{W} \cdot \ell_i)^{1-\gamma} \end{aligned} \tag{A.10}$$

Some algebra confirms the familiar growth rate formula

$$g = \frac{L^{\gamma}}{A^{\phi}} \cdot \frac{\int_0^1 \zeta_i \cdot \theta_i^{\frac{1}{1-\gamma}} \cdot (1+\Delta_i)^{-\frac{\gamma}{1-\gamma}} \cdot di}{\left(\int_0^1 \theta_i^{\frac{1}{1-\gamma}} \cdot (1+\Delta_i)^{-\frac{1}{1-\gamma}} \cdot di\right)^{\gamma}}. \tag{A.11}$$

Thus, the growth rate abstracting from the product-line level heterogeneity recovers the growth rate under full heterogeneity under the proposed measurement approach.

**Abundant resources.** Suppose aggregate supply of $L_t$ responds to productivity adjusted wage $W_t$ such that

$$L_t = \bar{L}_t \cdot \left(\frac{W_t}{Y_t}\right)^{\frac{\xi}{1-\gamma}}, \tag{A.12}$$

where $\bar{L}_t$ is given exogenously and $\xi/(1-\gamma)$ is the aggregate supply elasticity. Also, let $L_t^*$ be the supply in absence of frictions, i.e., when the R&D wage is at its frictionless level.

**Proposition 5.** *Under equations (2)-(5) and (A.12), we can express the economic growth rate in a Competitive Growth Equilibrium using the sample decomposition as in Proposition 1 with two adjustments. First, the frontier growth rate $g_t^F$ reflects the frictionless R&D input supply,*

$$g_t^F = \frac{L_t^{*\gamma}}{A_t^{\phi}} \cdot \left(\int_0^1 (\theta_{it} \cdot \zeta_{it})^{\frac{1}{1-\gamma}} \, di\right)^{1-\gamma}, \tag{A.13}$$

*and, second, R&D efficiency also reflects the potential effect on labor supply*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}} di}{\left(\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} di\right)^{\gamma}} \cdot \left(\int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} \cdot di\right)^{\frac{\xi \cdot \gamma}{1+\xi}}. \tag{A.14}$$

*Note that the supply elasticity only appears in the second term, which depends on the productivity-weighted average level of frictions. Any change in frictions or policy that keeps constant this average thus has the same effect on growth as in the case of $\xi = 0$.*

*Proof.* The proof follows from the same steps as in the derivation of the baseline results. □

Note, however, that the adjusted formulas tend to be less sensitive to variation in R&D returns. Intuitively, with flexible labor supply, excess demand for R&D workers tends to lead to more aggregate R&D employment instead of crowding-out demand from other firms.

**Proposition 6.** *Suppose that R&D returns, impact-value factors, and R&D productivity are jointly log-normally distributed and that R&D returns and impact-value factors are either positively or uncorrelated. Then, R&D efficiency is declining in the dispersion of log-R&D wedges as long as the supply of R&D inputs is sufficiently inflexible:* $\frac{1}{\gamma} > \frac{\xi}{1-\gamma}$*. Furthermore, holding constant the average level of R&D wedges, the Impact of R&D wedges is declining in the dispersion of R&D wedges as long as* $\gamma \geq \frac{\xi}{1+2\xi}$*.*

*Proof.* Solving for $\Xi_t$ under log-normal distribution and setting $\mu_\Delta = 0$, we have

$$\ln \Xi_t = -\frac{1}{2} \cdot \frac{\gamma}{(1-\gamma)^2} \cdot \left(\gamma - \frac{1}{1+\xi}\right) \cdot \sigma_\Delta^2.$$

It is straight-forward to show that this term is decreasing in $\sigma_\Delta^2$ iff $\frac{1}{\gamma} > \frac{\xi}{1-\gamma}$. Alternatively, setting $\mu_\Delta = -\frac{1}{2}\sigma_\Delta^2$ to maintain the average level of $1 + \Delta_{it}$, we have

$$\ln \Xi_t = -\frac{1}{2} \cdot \left(\frac{\gamma}{1-\gamma} - \frac{\xi}{1+\xi}\right) \cdot \sigma_\Delta^2,$$

which is declining in $\sigma_\Delta^2$ as long as the condition in the proposition holds. □

Importantly, aggregate estimates suggest that $\frac{\xi}{1-\gamma}$ is around 0.5 and, thus, easily satisfies the more stringent constraint given the consensus value for $\gamma = 0.5$ (Chetty et al., 2012).

**Free Entry.** Suppose that the mass $M_t$ of innovative firms is potentially responsive to changes in the economic environment and let $M_t^*$ be the mass of firms in absence of frictions. The equilibrium wage satisfies

$$\frac{W_t}{\gamma} = \left(\frac{M_t}{L_t}\right)^{1-\gamma} \left(\int_0^1 \theta_{it}^{\frac{1}{1-\gamma}} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} di\right)^{1-\gamma}. \tag{A.15}$$

I assume that all firm-types are permanent and that frictions $\Delta_{it}$ show up directly in the firm's cost function. The current period profits of an innovative firm are given by

$$\begin{aligned} \pi_{it} &\equiv \max \{\theta_{it} \cdot \ell_{it}^\gamma - W_t \cdot \ell_{it} \cdot (1+\Delta_{it})\} \\ &= (1-\gamma) \cdot \theta_{it}^{\frac{1}{1-\gamma}} \cdot ((W_t/\gamma) \cdot (1+\Delta_{it}))^{-\frac{\gamma}{1-\gamma}}. \end{aligned}$$

Assuming a constant discount factor and permanent types implies that current and expected, discounted value are proportional by factor $R/(R-1)$, where $R$ is the discount rate. The expected value of being an R&D firm is then given by

$$\begin{aligned}
\mathcal{V}_t &= \mathbb{E}_t\left[\frac{R}{R-1}\cdot\pi_{it}\right] \\
&= \frac{R}{R-1}\cdot(1-\gamma)\cdot\frac{\int_0^1 \theta_{it}^{\frac{1}{1-\gamma}}\cdot(1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}}\cdot di}{(W_t/\gamma)^{\frac{\gamma}{1-\gamma}}} \\
&= \frac{R\cdot(1-\gamma)}{R-1}\cdot\left(\frac{L_t}{M_t}\right)^{\gamma}\cdot\left(\int_0^1 \theta_{it}^{\frac{1}{1-\gamma}}\cdot di\right)^{1-\gamma}\cdot\frac{\int_0^1 \omega_{it}\cdot(1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}}\cdot di}{\left(\int_0^1 \omega_{it}\cdot(1+\Delta_{it})^{-\frac{1}{1-\gamma}}\cdot di\right)^{\gamma}}
\end{aligned}$$

I then assume that entry is such that entrants draw the attributed of a random firm among the existing distribution, implying expected value $\mathcal{V}_t$ and in turn need to pay entry cost. I consider two alternatives. In the first case, entry costs are in units of the output and given by $\phi_t^E\cdot\frac{R\cdot(1-\gamma)}{R-1}\cdot M_t^{\frac{\gamma}{\varphi}}$. The free entry condition is

$$\mathcal{V}_t = \phi_t^E\cdot\frac{R\cdot(1-\gamma)}{R-1}\cdot M_t^{\frac{\gamma}{\varphi}}$$

Using the formula for value of entry, we can then solve for equilibrium entry:

$$\frac{M_t}{M_t^*} = \left(\frac{\int_0^1 \omega_{it}(1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}}di}{\left(\int_0^1 \omega_{it}(1+\Delta_{it})^{-\frac{1}{1-\gamma}}di\right)^{\gamma}}\right)^{\frac{1}{\gamma}\frac{\varphi}{1+\varphi}} \quad \text{s.t. } M_t^* = \left(\frac{L_t}{\phi_t^{E\frac{1}{\gamma}}}\left(\int_0^1 \theta_{it}^{\frac{1}{1-\gamma}}di\right)^{\frac{1-\gamma}{\gamma}}\right)^{\frac{\varphi}{1+\varphi}}. \tag{A.16}$$

Note that $\varphi\to 0$ recovers the baseline model with $M_t = 1$, while $\varphi\to 0$ yields a standard free entry condition. In general, larger values of $\varphi$ make the mass of firms more responsive to the economic environment.

In the second case, I assume that entry cost are linked to the R&D wage and given by $\phi_t^E\cdot(1-\gamma)\cdot M_t^{\frac{1}{\varphi}}\cdot\frac{W_t}{\gamma}$. The free entry condition is

$$\mathcal{V}_t = \phi_t^E\cdot\frac{R\cdot(1-\gamma)}{R-1}\cdot M_t^{\frac{1}{\varphi}}\cdot\frac{W_t}{\gamma}$$

Using the formula for value of entry, we can then solve for equilibrium entry:

$$\begin{aligned}
\frac{M_t}{M_t^*} &= \left(\frac{\int_0^1 \omega_{it}(1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}}di}{\left(\int_0^1 \omega_{it}(1+\Delta_{it})^{-\frac{1}{1-\gamma}}di\right)^{\gamma}}\cdot\left(\int_0^1 \omega_{it}(1+\Delta_{it})^{-\frac{1}{1-\gamma}}di\right)^{\gamma-1}\right)^{\frac{\varphi}{1+\varphi}} \\
\text{s.t. } M_t^* &= \left(\frac{L_t}{\phi_t^E}\right)^{\frac{\varphi}{1+\varphi}}.
\end{aligned} \tag{A.17}$$

**Proposition 7.** *Under equations (2)-(5) and (A.16) or (A.17), we can express the economic growth rate in a Competitive Growth Equilibrium using the sample decomposition as in Proposition 1 with two adjustments. First, the frontier growth rate $g_t^F$ reflects frictionless entry,*

$$g_t^F = \frac{L_t^\gamma}{A_t^\phi} \cdot M_t^{*1-\gamma} \cdot \left( \int_0^1 (\theta_{it} \cdot \zeta_{it})^{\frac{1}{1-\gamma}} \, di \right)^{1-\gamma}, \tag{A.18}$$

*and, second, R&D efficiency also reflects potential effects on entry*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot (1+\Delta_{it})^{-\frac{\gamma}{1-\gamma}} di}{\left( \int_0^1 \omega_{it} \cdot (1+\Delta_{it})^{-\frac{1}{1-\gamma}} di \right)^\gamma} \cdot \frac{M_t}{M_t^*}, \tag{A.19}$$

*where $M_t/M_t^*$ is given by the respective formulas.*

*Proof.* The proof follows the same steps as before apart from taking the number of firms as a variable and using the entry condition. □

Private frictions now have an additional detrimental effect on growth through the number of firms. Notably, the entry-effect does not depend on the impact-value factor, which is irrelevant to firms' decision to enter or exit the economy. It is straight-forward to show that the impact of frictions is always worse in the economy with free entry holding constant the average level of R&D returns.

# B Data and Measurement Appendix

## B.1 Measurement

**Mapping patents to firms.** I assign patents to firms based on the crosswalk between patents and `PERMNO`s in Kogan et al. (2017), which I extend to `GVKEY`s using the mapping provided by WRDS.

**Measuring inventor employment.** Let $\mathcal{P}_{it\to t+4}$ be the set of successful patent applications for firm $i$ between $t$ and $t+4$ and $\mathcal{I}_{it\to t+4}$ be the set of associated inventors. I denote the number of patents assigned to firm $i$ and listing $j$ as inventor at time $t$ as $P_{ijt}$ and the total number of patents listing $j$ as inventor as $P_{jt}$. My measure of inventors is then given by

$$\texttt{Inventors}_{it\to t+4} = \sum_{j\in\mathcal{I}_{it\to t+4}} \frac{\sum_{s=0}^{4} P_{ijt+s}}{\sum_{s=0}^{4} P_{jt+s}}. \tag{B.1}$$

I use two additional measure in robustness checks. Firstly, I use the raw size of $|\mathcal{I}_{it\to t+4}|$, which forgoes the full-time equivalent adjustment, and, secondly, I construct the measure first at the 1-year horizon and then aggregate over the 5-year window. Note that the former is identical to the main measure when all inventors are only listed on patents that are also assigned to the firm.

**Return on Capital.** Following David (2021), I measure the return on capital as the ratio of sales to beginning of period capital stock. As for the R&D return, I construct the measure at the 5-year level:

$$\text{Return on Capital}_{it} \equiv \frac{\sum_{s=0}^{4} \texttt{Sales}_{it+s}}{\sum_{s=0}^{4} \texttt{Capital}_{it+s}}. \tag{B.2}$$

**Tobin's Q.** I define the (physical) investment Q as the ratio of firm valuation to physical capital (`ppeqgt`). I calculate firm valuation as stock price times outstanding shares plus debt net of cash holdings ($\texttt{prcc\_f} \times \texttt{csho} + \texttt{dltt} + \texttt{dlc} - \texttt{act}$).

**Liquidity.** I define liquidity as cash holdings divided by assets `ch/at`.

**Net-leverage.** I define net-leverage as short- and long-term minus net current assets divided by total assets $((\texttt{dlc} + \texttt{dltt}) - (\texttt{act} - \texttt{lct}))/\texttt{at}$.

**Dividend rate.** I define the dividend rate as dividends over assets `dvt/at`.

**Investment tax credits.** I normalize investment tax credits by capital investment `itci/capxv`.

**Knowledge intensity.** I define knowledge intensity as the ratio of knowledge capital divided by knowledge capital plus physical capital (`ppeqgt`). The knowledge capital measure is provided by Ewens et al. (2022).

**Profit rate.** I calculate the profit rate as earnings before extraordinary items (`ib`) divided by revenue (`revt`), both of which I first aggregate at the 5-year horizon.

**Public patent involvement.** I identify patents as connected to public actors either (a) if they are assigned to a government entity, research lab, or university or (b) if they have a government interest statement. I calculate the public value share as the ratio of patent valuations connected to public actors to total patent valuations for the relevant 5-year window.

**Firm dominance.** I construct firm dominance over inventors in two steps. First, for each of a firm's new patent within a 5-year window, I calculate the share of inventors working for the firm among those that worked on patents in the exactly same technology classification. For the latter, I use the complete CPC classification of the patent, which has more than 600 technology classes, which are non-exclusive at the patent level. Patents of the same technology class are thus those that have exactly the same classifications as the patent in consideration. As before, I distinguish between inventors using the USPTO disambiguation and link inventors to a firm if they are listed on a firm's new patent for the 5-year window in consideration. Second, I aggregate the patent-based measure to the firm-level by taking a simple average over the firm's new patents. Note that the resulting measure is between 0 and 1 by construction with 1 implying maximal dominance and vice versa.

**Inventor specialization.** I construct inventor specialization in two steps. First, I calculate inventor specialization for a given 5-year window as the average cosine similarity between patent classifications in an inventors portfolio of new patents. I rely on CPC classifications of patents, which has more than 600 non-exclusive patent categories. For each patent I then create an indicator vector over the set of available patent classification, where I weight individual categories by their inverse frequency. I then calculate the average cosine similarity across all patents in the portfolio and take the simple average across all patents. This measure is between 0 and 1 by construction with 0 implying completely different patents and 1 implying that all patents have the same technology classification. Second, I aggregate this measure up to the firm-level by taking a patent-weighted average across inventor associated with a firm, where the weight reflect the number of new patents shared by the inventor and firm. I interpret a larger value in this measure as more specialized inventors and vice verse following the logic that specialized inventors work on similar patents.

# C Empirical Appendix

## C.1 Additional Evidence

Table C.1: R&D Return Dispersion Across Specifications

| Specification | Standard Deviation | Observations |
|---|---|---|
| *A. Aggregation horizon* | | |
| 1-year | 1.00 | 11,083 |
| **5-year** | 0.93 | 11,845 |
| 10-year | 0.92 | 11,845 |
| 20-year | 0.91 | 11,845 |
| *B. Realization horizon* | | |
| Same year | 0.86 | 10,885 |
| **1-year** | 0.93 | 11,845 |
| 2-year | 0.97 | 10,852 |
| 5-year | 1.08 | 8,377 |
| *C. Minimum Patents* | | |
| **50 patents** | 0.93 | 11,845 |
| 100 patents | 0.84 | 7,846 |
| 200 patents | 0.79 | 4,859 |

*Note:* All return measures residualized with respect to NAICS3×Year fixed effects. Aggregation horizon refers to the number of years over which valuations and R&D expenditure are summed. Realization horizon refers to the difference between the year in which patents are filed and the year of R&D expenditure considered. Unless otherwise specified, R&D returns are measured with a 5-year aggregation horizon, 1-year realization horizon, and 50 minimum patents. Specifications in bold are the baseline. Returns are measured in logs.

Table C.2: R&D Return Dispersion Across Industries

| Industry | Standard Deviation | Observations |
|---|---|---|
| All industries | 0.93 | 11,844 |
| Life Science | 0.82 | 1,630 |
| IT | 1.07 | 4,732 |
| Manufacturing | 0.83 | 5,108 |
| Other | 0.83 | 374 |

*Note:* R&D return measures residualized with respect to NAICS3×Year fixed effects. Returns are measured in logs. Industries are defined as in Mezzanotti and Simcoe (2023). See text for detail.

Table C.3: Relationship of R&D Wedges and Impact-Value Factors over Time

| Impact-Value Factor | Estimate | Standard Error | $R^2$ | Observations |
|---|---|---|---|---|
| *A. Estimated Markup* | | | | |
| 1975–2014 | 0.030*** | (0.007) | 2.7% | 10,615 |
| 1975–1990 | 0.018*** | (0.005) | 3.1% | 3,182 |
| 2000–2014 | 0.034*** | (0.011) | 2.6% | 4,913 |
| *B. Profit-implied Markup* | | | | |
| 1975–2014 | 0.066*** | (0.016) | 4.6% | 11,845 |
| 1975–1990 | 0.028*** | (0.005) | 4.3% | 3,340 |
| 2000–2014 | 0.079*** | (0.017) | 5.8% | 5,469 |
| *C. Citations/Valuations* | | | | |
| 1975–2014 | -0.490*** | (0.034) | 16.4% | 11,845 |
| 1975–1990 | -0.497*** | (0.056) | 18.0% | 3,340 |
| 2000–2014 | -0.430*** | (0.045) | 12.0% | 5,469 |
| *D. **Citations/Δ Sales*** | | | | |
| 1975–2014 | -0.077 | (0.054) | 0.3% | 11,688 |
| 1975–1990 | -0.095 | (0.083) | 0.4% | 3,323 |
| 2000–2014 | -0.096 | (0.071) | 0.4% | 5,375 |
| *E. Citations/Valuations** | | | | |
| 1975–2014 | -0.201*** | (0.032) | 3.8% | 11,688 |
| 1975–1990 | -0.112* | (0.057) | 1.4% | 3,323 |
| 2000–2014 | -0.245*** | (0.040) | 5.6% | 5,375 |
| *F. Text-Impact/Δ Sales* | | | | |
| 1975–2014 | -0.184*** | (0.053) | 1.9% | 7,481 |
| 1975–1990 | -0.157** | (0.075) | 1.3% | 3,323 |
| 2000–2014 | -0.387*** | (0.076) | 9.0% | 1,168 |

*Note:* Each coefficient stems from a separate regression with the R&D wedge as the independent variable and a measure of the impact-value factor as the dependent variable. The R&D wedge is measured as the ratio of patent valuations over R&D expenditure except for the third row, where it is measured as changes in sales over R&D expenditure. Profit-implied markups are measured as one over one minus the profit rate, which is the ratio of profits to sales. Measured markups refer to the $\mu_2$ markup measure from Loecker et al. (2020). Variables are aggregated to the 5-year level as described in the text and the regression specification is in logs. All regressions control for NAICS3×Year fixed effects and standard errors are clustered at the NAICS6 level.

## C.2 Measurement Robustness

Table C.4: Return Dispersion with Adjustments

| Adjustment | Standard Deviation | Observations |
|---|---|---|
| *A. Acquisitions* | | |
| **Unadjusted** | 0.923 | 11,829 |
| Adjusted ($s = 6.3\%$) | 0.910 | 11,829 |
| Adjusted ($s = 8.5\%$) | 0.909 | 11,829 |
| Adjusted ($s = 100\%$) | 0.982 | 11,829 |
| *B. Fixed-costs* | | |
| **Unadjusted** | 0.924 | 11,807 |
| Adjusted | 0.937 | 11,807 |
| *C. Knowledge capital* | | |
| **R&D Expenditure** | 0.925 | 11,845 |
| Knowledge capital | 0.961 | 11,845 |
| Organizational capital | 0.985 | 11,845 |

*Note:* See text for description of measures. All return measures residualized with respect to NAICS3×Year fixed effects. Second column reports standard deviation of log R&D returns.

**Acquisitions.** Acquisitions are common in the innovation economy (Phillips and Zhdanov, 2013; Fons-Rosen et al., 2023). They might lead to measurement error in R&D returns due to misattribution, i.e., by not counting all R&D costs associated with the measured patents. Suppose that the firm acquires some inventions that are subsequently patented and added to total value created $z_{it} \cdot V_{it}$, however, the costs are recorded as acquisition cost $\text{Aqc.}_{it}$ instead of R&D expenditure $\text{R\&D}_{it}$. Assuming that the firm is otherwise unconstrained, the measured R&D return then becomes

$$\frac{V_{it} \cdot z_{it}}{\text{R\&D}_{it}} = \frac{1}{\gamma} \cdot \left(1 + \frac{\text{Aqc.}_{it}}{\text{R\&D}_{it}}\right),$$

which may yield measured R&D return dispersion, to the degree that acquisition intensities differ across firms, even though true R&D returns are equalized.

I propose the following approach to investigating the importance of acquisitions for R&D return dispersion. First, I assume that firms use a fixed fraction $s$ of total reported acquisition expenditure on innovative products such that $\text{Aqc.}_{it} = s \cdot \text{Total aqc.}_{it}$. Note that total acquisition expenditure is reported in Compustat. Second, assuming that the acquisition intensity is relatively small, we can estimate $s$ as the semi-elasticity of R&D returns with respect to the total acquisition intensity using OLS. I find $s \in \{6.3\%, 8.6\%\}$ depending on the precise fixed effects. Finally, we can construct adjust R&D returns, which should be equalized across firms, as

$$\frac{V_{it} \cdot z_{it}}{\text{R\&D}_{it} + \hat{s} \cdot \text{Total aqc.}_{it}} = \frac{1}{\gamma}.$$

Panel A of Table C.4 reports the associated results. Adjusting for acquisitions marginally reduces R&D return dispersion, however, the magnitudes are small. Adjusting by 8.5% of total acquisitions reduces measured R&D return dispersion by 1.5%. Counting all acquisitions as R&D expenditure increases R&D return dispersion. Thus, acquisition, as captured by this adjustment, do no appear to be a significant driver of R&D return dispersion.

**Fixed costs of R&D.** Suppose that firms face fixed R&D costs $f_i \cdot W_t$. Then, total R&D expenditure is given by $(f_i + \ell_{it}) \cdot W_t$ and the R&D return in absence of frictions by

$$\frac{V_{it} \cdot z_{it}}{(f_i + \ell_{it}) \cdot W_t} = \frac{1}{\gamma} \cdot \frac{\ell_{it}}{f_i + \ell_{it}}. \tag{C.1}$$

Resultingly, as long as firms face some fixed-costs, their average R&D return will be increasing in their quantity of R&D conducted $\ell_{it}$ and we have R&D return dispersion that is unrelated to frictions. Note, however, that the average R&D return for very large firms, i.e. $\ell_{it} >> f_i$, is still approximately constant.

I propose a simple approach to investigate the importance of fixed costs. First, I assume that fixed costs are identical within a NAICS3×5-Year cell. Second, let $\bar{\Delta}$ be the average R&D return for a in the top 75th percentile and $\underline{\Delta}$ be the average R&D return for a firm in the 25th percentile. I can then estimate the industry specific $\hat{\gamma}$ as inverse of the average R&D return for firm in or above the 75th percentile of R&D expenditure. Finally, let $\underline{TC}$ be the average total R&D expenditure of a firm in 25th percentile of the R&D cost distribution. I can then estimate fixed costs and adjusted R&D returns as

$$\hat{f}_i \cdot W_t = \underline{TC}_i \cdot \left(1 - \frac{\underline{\Delta}_i}{\bar{\Delta}_i}\right) \quad \text{and} \quad \frac{V_{it} \cdot z_{it}}{TC_{it} - \hat{f}_i \cdot W_t} = \frac{1}{\gamma}.$$

The measure will estimate larger fixed costs if firms with high R&D expenditure also tend to have much larger R&D returns and vice versa. The corrected R&D returns should be equalized across firms.

Panel B of Table C.4 reports the associated results. The fixed-costs adjustment increases measured R&D return dispersion marginally. Thus, fixed-costs, as captured in this adjustment, do appear to be a significant source of measured R&D return dispersion.

**Knowledge capital.** A growing literature in economics and finance interprets R&D expenditure as a cumulative investment in the firms knowledge base (Peters and Taylor, 2017; Tambe et al., 2019; Crouzet and Eberly, 2023). Under this alternative view, R&D capital is the appropriate denominator for the R&D returns instead of R&D expenditure. I explore the robustness of my findings with respect to the input measure using the knowledge capital and organizational capital measures developed in Ewens et al. (2022). The knowledge capital measure is built up from R&D investments only, while organizational capital focuses on other overhead expenses. I refer to the sum of both as organizational capital.

Panel C of Table C.4 reports the associated results. R&D return dispersion is strictly higher when using either the knowledge capital or organizational capital. R&D return dispersion is thus robust to alternative measure of R&D inputs as captured in these adjustments.

**Outlier patents.** Innovation outcomes are famously fat-tailed (Akcigit and Kerr, 2018). While most inventions have a moderate impact at best, some, like the iPhone, transform entire industries. This consideration raises the question as to how much of the variation in R&D returns is driven by "outlier-patents" with extremely large valuations. I investigate this question by creating winsorized measures of patent valuations that ignore value above the top 1% or top 5% of the annual patent valuation distribution and recalculate R&D return dispersion. As reported in Panel A of Table C.5, winsorizing patent valuations at the top 1% (5%) reduces R&D return dispersion by 1% (3.6%). Thus, only a small fraction of the dispersion in R&D returns is potentially attributable to outlier patents.

**Low value patents.** Jaffe and Lerner (2007), among others, argue that changes in patent law, grant procedures, and enforcement have led to an onslaught of low quality patents with questionable economic value. While the methodology in Kogan et al. (2017) does take into account low quality patents, we might still wonder whether their presence adds more noise to measured R&D returns.[28] I investigate this question by constructing measures excluding valuations below 250k (500k) in 2010 USD and recalculating R&D return dispersion.[29] As reported in Panel B of Table C.5, excluding low quality patents from the measure increases measured dispersion in R&D returns slightly.

[28] The measure developed in Kogan et al. (2017) yields strictly positive and monotonically increasing patent valuation as a function of the the stock market return. Thus, even if the stock market does not respond at all, because the patent is worthless, they will assign a positive value to the patent.

[29] In line with the hypothesis that the share of low quality patents has increased, I find that 10% (15%) of patents are valued less than 250k (500k) from 1975-84, while 17% (22%) are for the 2005-14 period.

Table C.5: Return Dispersion with Patent Valuation Adjustments

| Adjustment | Std. dev. of R&D return | Observations |
|---|---|---|
| *A. Outlier patents* | | |
| **Unadjusted valuations** | 0.925 | 11,845 |
| Winsorized at top 1% | 0.916 | 11,845 |
| Winsorized at top 5% | 0.892 | 11,845 |
| *B. Low value patents* | | |
| **All valuations** | 0.925 | 11,845 |
| Valuations > 250k | 0.942 | 11,845 |
| Valuations > 500k | 0.959 | 11,842 |
| *C. Class grant rate* | | |
| **Unadjusted** | 0.925 | 11,845 |
| Adjusted | 0.933 | 11,845 |
| *D. Value-dependent grant rate* | | |
| $\eta = 0$ **(Unadjusted)** | 0.925 | 11,845 |
| $\eta = .05$ | 0.968 | 11,845 |
| $\eta = .25$ | 1.163 | 11,845 |
| $\eta = .5$ | 1.740 | 11,845 |
| $\eta = 1.5$ | 2.584 | 11,845 |
| $\eta = 2$ | 1.648 | 11,845 |

*Note:* See text for description of measures. All return measures residualized with respect to NAICS3× Year fixed effects. Second column reports standard deviation of log R&D returns.

**Measurement details of patent valuations.** Kogan et al. (2017) measure patent valuations using the idea that a patent grant should increase the value of the firm by the unexpected part of the patent value. Let $M$ be the valuation of the firm, $V$ be the value of the patent, and $\pi$ the ex-ante probability of the patent being granted, then the change in firm valuation $\Delta M$ at the moment that the patent is granted should equal

$$\Delta M = (1 - \pi) \cdot V \quad \text{or, equivalently,} \quad V = \frac{\Delta M}{1 - \pi}. \tag{C.2}$$

They measure the nominator using stock market returns and assume that the probability that a patent is granted is constant across all patents. The latter assumption is quite stringent for at least two reasons. First, patents of different patent classes might have different probabilities of being granted. For example, during the 1991-2014 period, 85% of patents applications classified as semiconductor memory devices (CPC subclass G11C) were granted within 3.5 years compared to 30% of those classified as healthcare informatics (CPC subclass G16H). Second, patent grants are assumed to be independent of the value of the patent. Such an assumption would not hold, e.g., if higher quality patents are more valuable, but also more likely to be granted.

I investigate whether either of these possibilities contributes to R&D return dispersion as follows. First, I use data on patent application and grant decision from the USPTO for the 1991–2014 period to calculate the grant probability by patent class and construct an adjusted valuation that takes into account differences in grant probabilities. For a given CPC subclass, I calculate the grant probability as the share of patents that were granted within 3.5 years of the application. Second, I assume that the probability of patent rejection takes the form $1 - \pi_p = \pi_0 \cdot V_p^{-\eta}$, where $\eta$ measures the degree to which higher value patents are also more likely to be granted. The adjusted patent valuation is then given by

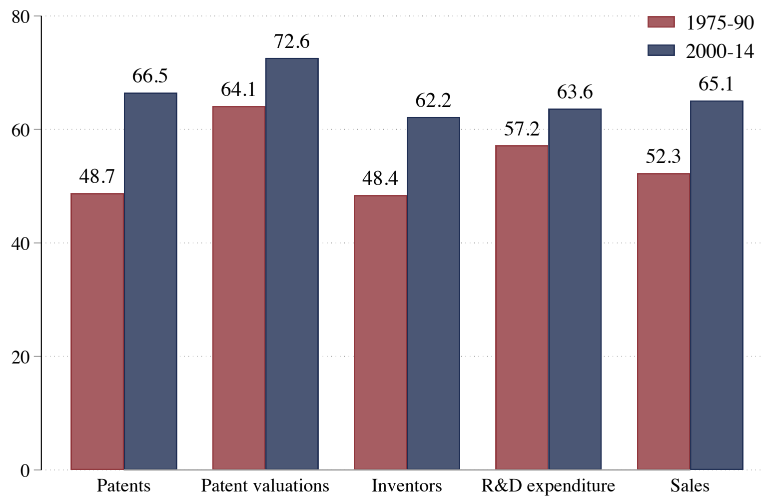
$$\tilde{V}_p = \left( V \cdot \frac{1 - \bar{\pi}}{\pi_0} \right)^{\frac{1}{1-\eta}} . \tag{C.3}$$

I calibrate $\pi_0$ at the annual level to keep the average patent valuation constant and experiment with alternative values for $\eta$. Optimally, one would want to estimate this value, however, we only observe valuations for granted patents. Finally, note that for $\eta > 1$, the ranking of patent valuations is reversed such that patents that are more valuable according to Kogan et al. (2017) are less valuable according to the adjusted measure.

I find that neither adjustment reduces the measured dispersion as reported in Panel C and D of Table C.5. Panel C considers the case of adjustment for class grant probabilities and shows that R&D return dispersion is marginally larger when taking them into account. Panel D considers value-dependent grant rates and find that, depending on the assumed value-grant rate elasticity, measured R&D return dispersion is much larger when taking them into account. For example, it increases by 25% when assuming that a 10% larger patent valuation translated into a 2.5% higher probability of being granted.

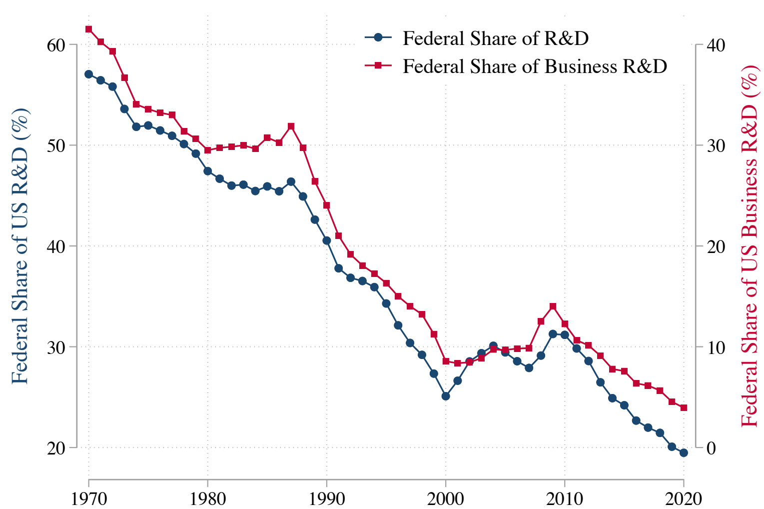
## C.3 Potential Drivers of Slower Growth

Figure C.1: Concentration Is Rising in the R&D Sector




*Notes:* Share for firms in top 5% of total for respective periods in sample. See text and Appendix for variable description.

Figure C.2: Federal Support for R&D Has Decline Since 1975

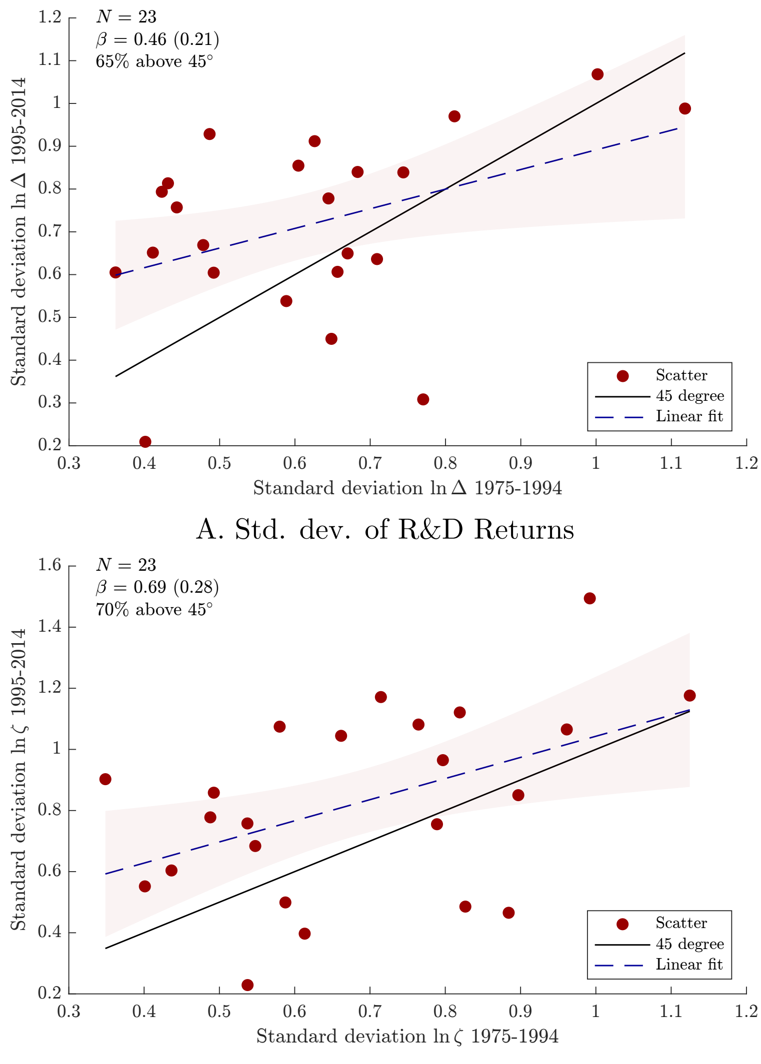



*Notes:* Author's calculations based on NSF National Pattern.

## C.4 Cross-Industry Comparisons

Figure C.3 investigates the persistence of key summary statistics across time. Each observation is a NAICS4 industry with at least 25 active firms in the early and late period. The figures then plot scatters of a given summary statistic in the first half against the second half of the sample together with a linear fit and the 45 degree line.

Figure C.3: Within-Industry Summary Measures Across Time




A. Std. dev. of R&D Returns

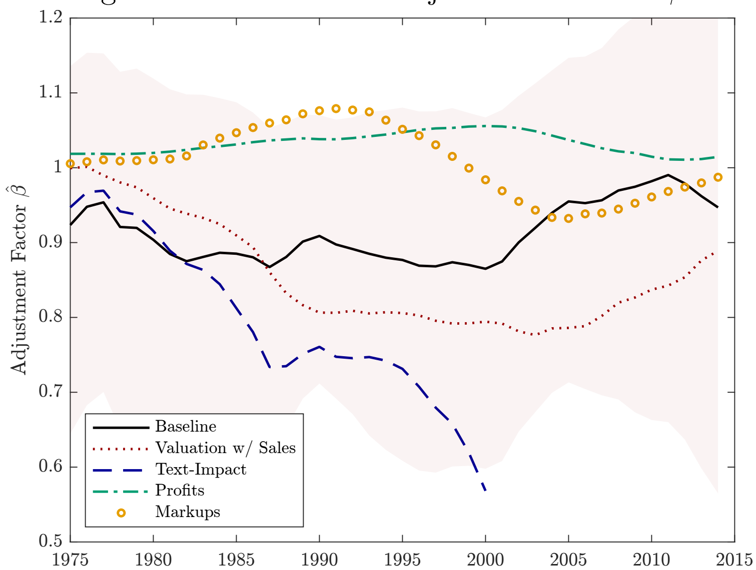


B. Std. dev. of Impact-Value Factors

*Notes:* Figures show scatter plots and regression lines for persistence of within-industry summary measures across time. Each dot represents a 4-digit industry and the sample if restricted to industries with at least 50 observations within a period.

## C.5 Annual Adjustment Factors

Figure C.4: Estimated Adjustment Factors $\hat{\tilde{\beta}}$




*Notes:* This figure reports annual estimates for adjustment factor $\tilde{\beta}$ depending on alternative proxies for the impact-value factor. Each estimate is calculated as $\sqrt{1 + 2 \cdot \hat{\beta}}$, where $\hat{\beta}$ is the regression coefficient of log R&D returns when regressed on the respective proxy for the log impact-value factor. Annual estimates are calculated using a 10-year centered rolling window. See text and Appendix for details.

## C.6 Constructing Aggregate Estimates for Continuing Firms

A natural question in light of the evolution of R&D efficiency is whether entry and exit contributed significantly. To shed light on this question, I construct a measure of R&D efficiency that is concerned with continuing firms only. I construct the baseline measure for 1975 and measures of annual changes in the Impact of R&D wedges for all subsequent years, which I accumulate over time. For the year 1976, I first filter out all firms which are active in 1975 and 1976. For these firms, I calculate estimates of R&D efficiency for both 1975 and 1976. I then take the ratio to get the rate of change and apply it to my original estimate for 1975. Subsequent years are calculated accordingly.

In formulas, let $\hat{\Xi}_t$ be the baseline estimate for the Impact of R&D wedges for year $t$. Let $\hat{\Xi}_t^{t,t-1}$ be the estimate when using only firms that were active in both $t$ and $t-1$ with $\hat{\Xi}_{t-1}^{t,t-1}$ being the respective value for $t-1$. I then calculate the time-series for the Impact of R&D

wedges for continuing firms $\hat{\Xi}_t^C$ as

$$\hat{\Xi}_t^C = \begin{cases} \hat{\Xi}_t & \text{if} \quad t = 1975 \\ \hat{\Xi}_{t-1}^C \cdot \left( \frac{\hat{\Xi}_t^{t,t-1}}{\hat{\Xi}_{t-1}^{t,t-1}} \right) & \text{if} \quad t = 1976, ..., 2014 \end{cases} . \tag{C.4}$$

Differences in the evolution of this estimate versus the baseline estimate occur due to changes in the gap of the R&D wedge when calculated for all versus for continuing firms.

## C.7 Robustness for Aggregate Measures

Table C.6 reports estimates of R&D efficiency for alternative specifications.

Table C.6: R&D Wedges, Economic Growth and Welfare — Specification Robustness

| Specification | Growth Impact $\Xi - 1$ | | | | Welfare Cost of $\Delta$ | |
|---|---|---|---|---|---|---|
| | 1975–2014 | 1975–90 | 2000–14 | $\Delta$ | End. | Semi-End. |
| *A. Fixed Effects* | | | | | | |
| Year | -21.3% | -15.4% | -25.2% | -11.7% | 5.8% | 5.7% |
| **NAICS3 × Year** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| NAICS6 × Year | -13.5% | -8.1% | -17.6% | -10.3% | 5.1% | 5.0% |
| *B. Minimum Patents* | | | | | | |
| **50 Patent** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| 100 Patents | -17.3% | -10.7% | -21.7% | -12.3% | 6.2% | 6.0% |
| 200 Patents | -16.5% | -9.4% | -21.5% | -13.4% | 6.8% | 6.7% |
| *C. Time Horizon* | | | | | | |
| **5-Year** | -17.9% | -12.0% | -21.7% | -11.0% | 5.5% | 5.3% |
| 10–Year | -18.1% | -16.3% | -19.8% | -4.2% | 1.9% | 1.9% |
| 20–Year | -17.2% | -17.9% | -18.6% | -0.9% | 0.4% | 0.4% |

*Notes:* Table reports estimates of R&D efficiency across samples together with their implications for welfare. Changes in welfare are in consumption equivalent terms. See text and Appendix for details.

# D Measurement Error

This section considers adjustments for two sources of measurement error in R&D returns: Uncertainty across R&D projects within a firm and firm-level uncertainty in R&D outcomes. The former arises when firms conduct R&D projects whose ex-post value to the firm is uncertain, e.g., because some inventions turn out more valuable than others. The latter arises when there are firm-level shocks that affect the value of R&D outputs after investments are made, e.g., general taste shocks for the firm's products. I propose a bootstrapping procedure to address the former and a structural GMM approach to address the latter.

## D.1 Bootstrapping for Noise

Suppose the value of individual research projects, as represented by patents, is ex-ante uncertain. Ex-post variation in valuations then might give rise to dispersion in measured R&D returns even with equalized ex-ante expectations. I propose a simple bootstrapping procedure to estimate the variability in R&D returns induced by this variation.

I establish the realized portfolio of patent valuations for each firm $\times$ 5-year interval in which the firm has at least 50 patents. Throughout, I take the true number of patents that a firm has achieved as given. For each of 1000 bootstrap samples I then implement the following procedure:

1. For each firm and 5-year window in which the firm has at least 50 patents:
   (a) From the realized portfolio for the firm-period, draw with replacement an alternative portfolio with the same number of patents.
   (b) Calculate the return gap as the ratio as the log of valuations in the alternative portfolio divided by the valuation of the true portfolio.
2. Calculate the within-period standard deviation of return gaps for the simulated data.

One way to interpret this approach is that the realized patent portfolio is a good approximation for the true uncertainty faced by the firm around its innovation outcomes. The procedure ignores all variation coming from shifts in the level of expected patent valuation and instead considers the dispersion conditional on the average value only. As a result, the procedure will overstate the associated measurement error if firms are aware that certain project are low or high expected value within their research portfolio. On the other hand, the procedure ignores all uncertainty around the number of realized patents.

Table D.1 reports the estimates. I find an average standard deviation of the return gap of around 0.06, which suggests that uncertainty around patent valuation might have contributed $(0.06/1.1)^2 \approx 0.2\%$ of the variance of R&D returns. Uncertainty across patent, thus, did not appear to have a large contribution of dispersion therein. Note that this is not necessarily surprising, since averages should converge to the true mean with a sufficiently large number of independent observations by the law of large numbers.

Table D.1: Bootstrapping Estimates for Measurement Error

| Measure | Period | | |
|---|---|---|---|
| | 1975-2014 | 1975-1990 | 2000-2014 |
| Standard deviation | 0.059 | 0.056 | 0.058 |
| | [0.051,0.069] | [0.049,0.066] | [0.051,0.069] |
| Adjustment factor | 0.998 | 0.997 | 0.998 |
| | [0.997,0.998] | [0.996,0.998] | [0.997,0.999] |

*Notes:* Table reports bootstrapping estimates for noise in R&D returns. See text for details.

## D.2 GMM Approach

The bootstrapping approach can address variation across projects, however, it cannot adjust for correlated shocks to the firms' patent valuations or citations, which could arise, e.g., due to the expectation-realization gap, correlated errors in patent valuation estimation, or misreporting of R&D expenditure.[30] I propose to investigate the importance of such variation using a structural decomposition of the variation in R&D returns.

Consider a stationary, AR(1) process $\{y_{it}\}$:

$$y_{it} = (1-\rho)\mu_i + \rho y_{it-1} + \varepsilon_{it} \quad \text{with} \quad \varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \quad \text{and} \quad \mu_i \sim N(0, \sigma_\mu^2). \tag{D.1}$$

The econometrician observes the process with i.i.d. normal measurement error:

$$\tilde{y}_{it} \equiv y_{it} + \nu_{it} \quad \nu_{it} \overset{iid}{\sim} N(0, \sigma_\nu^2). \tag{D.2}$$

[30] Note that R&D expenditure is expensed in US GAAP accounting, giving firms an incentive to fully report R&D expenditure to reduce their tax liability. Terry et al. (2022) argue that managers still might misreport when attempting to hit short-run earnings targets or smooth earnings. See also Dukes et al. (1980); Baber et al. (1991); Lev et al. (2005); Chen et al. (2021); Terry (2023).

**Lemma 2.** *Define* $\Delta\tilde{y}_{it} \equiv \tilde{y}_{it} - \tilde{y}_{it-1}$*, then under* $\rho \in (0,1)$*, we have*

$$\begin{aligned} m_1 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it}) = \frac{1}{1+\rho}\sigma_\varepsilon^2 + \sigma_\nu^2 \\ m_2 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-1}) = \frac{\rho}{1+\rho}\sigma_\varepsilon^2 \\ m_3 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-2}) = \frac{\rho^2}{1+\rho}\sigma_\varepsilon^2 \\ m_4 &\equiv Cov(\tilde{y}_{i,t}, \tilde{y}_{it-1}) = \sigma_\mu^2 + \frac{\rho}{1-\rho^2}\sigma_\varepsilon^2. \end{aligned}$$

**Proposition 8.** *If* $\rho \in (0,1)$*, we can solve for* $\{\rho, \sigma_\mu, \sigma_\varepsilon, \sigma_\nu\}$ *using the population auto-covariance structure of* $\tilde{y}_{it}$ *and* $\Delta\tilde{y}_{it} \equiv y_{it} - y_{it-1}$*:*

$$\beta \equiv \begin{bmatrix} \rho \\ \sigma_\varepsilon^2 \\ \sigma_\mu^2 \\ \sigma_\nu^2 \end{bmatrix} = \begin{bmatrix} \frac{m_3}{m_2} \\ \frac{(m_2)^2}{m_3} + m_2 \\ m_4 - \frac{(m_2)^2}{m_2 - m_3} \\ m_1 - \frac{(m_2)^2}{m_3} \end{bmatrix}$$

*Let* $\Omega$ *be the covariance matrix of* $m$ *and denote the sample moments by* $\hat{m}$*, then*

$$\hat{\beta} \sim N(\beta, \Sigma) \quad \text{and a feasible estimator is} \quad \hat{\Sigma} = \left(\frac{\partial\hat{\beta}}{\partial m}\right)' \hat{\Omega} \left(\frac{\partial\hat{\beta}}{\partial m}\right),$$

*where* $\partial\beta/\partial m$ *is evaluated at* $\hat{m}$ *and given by*

$$\frac{\partial\beta}{\partial m} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -\frac{m_3}{(m_2)^2} & 2\frac{m_2}{m_3} + 1 & m_2\left(\frac{m_2 - 2m_3}{(m_2 - m_3)^2}\right) & -2\frac{m_2}{m_3} \\ \frac{1}{m_2} & -\left(\frac{m_2}{m_3}\right)^2 & -\left(\frac{m_2}{m_2 - m_3}\right)^2 & -\left(\frac{m_2}{m_3}\right)^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

*Proof.* The first part follows by rearranging the moments expressions. The second part follows from the Law of Large Numbers for the moment vector and the Delta method. □

I report estimates for two measures of R&D returns in Table D.2. I find no contribution of purely transitory shocks to the overall variation for my baseline measure of R&D returns in column 1, however, the estimates are somewhat imprecise. Using sales to measure R&D output yields a contribution of 22%.

Table D.2: GMM Parameter Estimates for AR(1) with Noise

| Parameter | Valuations/ R&D | $\Delta$Sales/ R&D |
|---|---|---|
| $\rho$ | 0.626*** | 0.787*** |
| | (0.083) | (0.149) |
| $\sigma^2_\epsilon$ | 0.628*** | 0.404*** |
| | (0.085) | (0.092) |
| $\sigma^2_\mu$ | 0.197 | −0.086 |
| | (0.140) | (0.726) |
| $\sigma^2_\nu$ | 0.007 | 0.509*** |
| | (0.090) | (0.091) |
| Observations | 8,014 | 7,653 |
| Adjustment factor | 0.997 | 0.810 |

*Notes:* Table reports parameters estimates for AR(1) with Noise in logs using a General Methods of Moments approach. See text for details.

## D.3 Adjustment Factors

I implement a measurement adjustment by calculating shrinkage factors according to the simple formula:

$$\beta = \sqrt{\frac{\sigma^2 - \sigma^2_\nu}{\sigma^2}}, \tag{D.3}$$

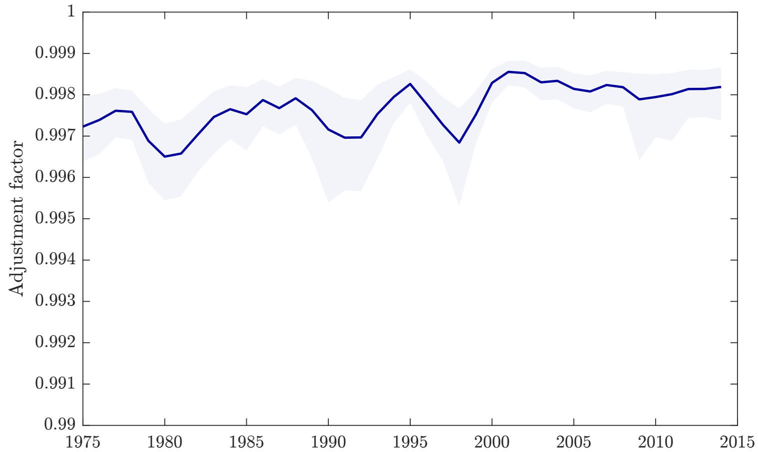
where $\sigma^2$ is the overall variation in R&D wedges and $\sigma^2_\nu$ is the estimated measurement error variance. The adjustment factor has the simple property that

$$Var(\beta \times \ln(1 + \Delta_{it})) = \beta^2 \times Var(\ln(1 + \Delta_{it})) = \frac{\sigma^2 - \sigma^2_\nu}{\sigma^2} \times \sigma^2 = \sigma^2 - \sigma^2_\nu. \tag{D.4}$$

The adjusted wedges then have the estimated true underlying variance.

Due to data limitation, I only calculate the adjustment factor for the GMM method for the full sample and for bootstrapping method on a year-by-year basis. Figure D.5 reports the estimated shrinkage factor for the bootstrapping method, which is small.

Figure D.5: Measurement Error Shrinkage Factors




*Notes:* Figure plots shrinkage factors for measurement error adjustment using alternative approaches.

# E Adjustment Costs and R&D Wedges

This section investigates the contribution of adjustment costs to R&D return dispersion. I introduce a dynamic heterogeneous firms growth model in line with the framework in the main text and calibrate it with fixed and quadratic adjustment costs via moment matching. Adjustment costs in the calibrated model can account for 16% of the empirical R&D return dispersion, but fail to capture their persistence in the data.

## E.1 Setup

**Household.** The representative household consists of a unit mass of workers, whereof $L_R$ work in research and $L_P = 1 - L_R$ in production. The household owns all firms and receives dividend income $\Pi_t$ in addition to wages $W_{R,t}$ and $W_{P,t}$. Income can either used for consumption $C_t$ or saved in riskless bond $B_t$ paying interest $R_t$. The riskless bond is in zero net-supply. Households have log preferences over consumption and discount the future with factor $\beta$. Their optimization problem is thus given by

$$U \equiv \max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \cdot \ln C_t. \qquad \text{s.t.} \quad B_{t+1} + C_t = R_t \cdot B_t + W_{R,t} \cdot L_R + W_{P,t} \cdot L_P + \Pi_t. \tag{E.1}$$

The optimization problem yields the standard Euler equation

$$\frac{C_{t+1}}{C_t} = \beta \cdot R_{t+1}. \tag{E.2}$$

**Production.** The final output in the economy is produced with production labor and intermediate inputs $\{x_{it}\}_{i \in A_t}$:

$$Y_t = L_P^{1-\alpha} \cdot \int_0^{A_t} x_{it}^{\alpha} \cdot di \tag{E.3}$$

Profit maximization yields the demand function for intermediate inputs:

$$p_{it} = \alpha \left( \frac{L_P}{x_{it}} \right)^{1-\alpha}. \tag{E.4}$$

Each intermediate input is owned by one of a unit mass of innovative firms. The firms face constant unit costs of production $\psi$ and set the price as to maximize profits:

$$\pi_{it} \equiv \max \{p_{it} \cdot x_{it} - \psi \cdot x_{it}\} \quad \text{s.t.} \quad (\text{F.6}). \tag{E.5}$$

The profit maximizing price is given by

$$p_{it} = \frac{\psi}{\alpha}. \tag{E.6}$$

I normalize this value to 1 by setting $\psi = \alpha$.

The resulting equilibrium profits are constant across all intermediate inputs and given by

$$\pi_{it} = \pi = (1-\alpha)\alpha^{\frac{1}{1-\alpha}} \cdot L_P. \tag{E.7}$$

Equilibrium output and net-output are given by

$$Y_t = A_t \cdot L_P \cdot \alpha^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad \tilde{Y}_t = Y_t - \psi \int_0^{A_t} x_{it} di = A_t \cdot L_P \cdot \alpha^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha^2). \tag{E.8}$$

Equilibrium wages for production workers satisfy

$$W_{P,t} = A_t \cdot (1-\alpha) \cdot \alpha^{\frac{\alpha}{1-\alpha}}. \tag{E.9}$$

**Innovation.** There is a unit mass of innovative firms that create new intermediate goods using R&D labor $\ell_{it}$ with idea production function

$$z_{it+1} = A_t \cdot \varphi_{it} \cdot \ell_{it}^{\gamma}. \tag{E.10}$$

R&D productivity $\varphi_{it}$ follows an AR(1) process in logs:

$$\ln \varphi_{it} = (1-\rho) \cdot \mu + \rho \cdot \ln \varphi_{it-1} + \epsilon_{it} \quad \text{with} \quad \epsilon_{it} \sim N(0, \sigma^2). \tag{E.11}$$

The value of an invention is the present discounted value of the associated profits:

$$\mathcal{V} = R^{-1} \cdot \sum_{t=0}^{\infty} R^{-t} \cdot \pi = \frac{1}{R-1} \cdot \pi. \tag{E.12}$$

The cost of R&D depend on the R&D wage $W_{R,t}$ and fixed and quadratic adjustment costs $\theta^F$ and $\theta^Q$, which are denoted in terms of the R&D wage for expositional convenience:

$$C_t(\ell_{it}, \ell_{it-1}) = W_{R,t} \cdot \left(1 + \theta^F \cdot A_t \cdot \{\ell_{it} \neq \ell_{it-1}\} + \theta^Q \cdot A_t \cdot \left(\frac{\ell_{it} - \ell_{it-1}}{\ell_{it-1}}\right)^2\right) \cdot \ell_{it} \quad \text{(E.13)}$$

Firms choose R&D input $\ell_{it}$ to maximize firm value:

$$V_t(\varphi_{it}, \ell_{it-1}) = \max_{\ell_{it}} \left\{ z_{it+1} \cdot \mathcal{V} - C_t(\ell_{it}, \ell_{it-1}) + R^{-1} \cdot \mathbb{E}_t[V_{t+1}(\varphi_{it+1}, \ell_{it})] \right\}. \quad \text{(E.14)}$$

The labor market clearing condition for R&D workers pins down their wage:

$$L_R = \int_0^1 \ell_{it} \cdot di. \quad \text{(E.15)}$$

Aggregate profits are given by the profits from production minus adjustment costs:

$$\Pi_t = A_t \cdot \pi - \int_0^1 W_{R,t} \cdot \ell_{it} \cdot \left(\theta^F \cdot A_t \cdot \{\ell_{it} \neq \ell_{it-1}\} + \theta^Q \cdot A_t \cdot \left(\frac{\ell_{it} - \ell_{it-1}}{\ell_{t-1}}\right)^2\right) \cdot di \quad \text{(E.16)}$$

Finally, the growth rate of the mass of intermediate goods is given by

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \int_0^1 \tilde{z}_{it+1} \cdot di, \quad \text{(E.17)}$$

where $\tilde{z}_{it} = z_{it}/A_t$.

**Equilibrium.** Finally, I define a competitive equilibrium and balanced growth path as is customary. Throughout, I assume that the economy is on its long-run balanced growth path.

**Definition 3** (Competitive Equilibrium (CE))**.** *For a given initial level* $\{A_0, \{\varphi_{i0}\}\}$ *and the R&D productivity process* $\{\varphi_{it}\}$ *in* (F.13)*, a CE is a sequence* $\{A_t, C_t, W_{R,t}, R_t, \{\ell_{it}, V_t(\ell_{it-1}, \varphi_{it})\}\}$ *such that households optimize, innovative firms choose R&D employment optimally, and markets clear.*

**Definition 4** (Balanced Growth Path (BGP))**.** *A BGP is a CE such that* $\{A_t, W_{R,t}, V_t(\cdot)\}$ *grow as a common rate* $g$ *and the interest rate* $R_t$ *is constant.*

**Balanced Growth.** We can normalize the value function along a BGP to get

$$v(\varphi_{it}, \ell_{it-1}) = \max_{\ell_{it}} \{\tilde{z}_{it+1} \cdot \mathcal{V} - C(\ell_{it}, \ell_{it-1}) + \beta \cdot \mathbb{E}_t[v(\varphi_{it+1}, \ell_{it})]\}$$
$$\text{with} \quad C(\ell_{it}, \ell_{it-1}) = \left(1 + \theta^F \cdot \{\ell_{it} \neq \ell_{it-1}\} + \theta^Q \cdot \left(\frac{\ell_{it} - \ell_{it-1}}{\ell_{it-1}}\right)^2\right) \cdot w_R \cdot \ell_{it}, \tag{E.18}$$

where $w_R = W_{R,t}/A_t$. This problem can be solved using conventional numerical methods.

## E.2 Calibration and Counterfactuals

I calibrate the 10 model parameters using a combination of external calibration and moment matching. I calibrate $\beta = 0.97$ to achieve a real interest rate close to 5% in light of a long-run growth rate of 1.5% and $\alpha = 0.8$ to get a markup of 25% as in Terry (2023). I set the R&D scale elasticity to $\gamma = 0.5$, which implies an R&D unit cost elasticity of -1 in absence of frictions and is standard in the literature (Acemoglu et al., 2018). Finally, I calibrate the supply of labor $\{L_R = 0.142, L_P = 1 - L_R\}$ such that 14.2% of the workforce are employed in R&D as in Acemoglu et al. (2018).

The remaining parameters are calibrated using moment matching as summarized in Table E.1.[31] I target the parameters of the R&D productivity process $\{\mu, \sigma, \rho\}$ with a long-term growth rate of 1.5%, the standard deviation of R&D productivity, and the autocorrelation of R&D productivity corrected for firm-fixed effects as in Han and Phillips (2010). I measure R&D productivity at the 5-year level as in the main text. Next, inspired by Asker et al. (2014), I target the adjustment cost parameters $\{\theta_F, \theta_Q\}$ with the share of observations with R&D growth less than 2.5% and more than 25% in absolute value, respectively. High fixed costs of adjustment increase the share of observations with no change in R&D and are thus directly linked to the former moment. High quadratic fixed costs increase the costs of large adjustments and are, thus, directly linked to the latter moment. I also add further moments describing the behavior of the growth rate of R&D investment including the change in probability of a small adjustment after seeing a large adjustment, which is closely related to the behavior of R&D under fixed adjustment costs, the ratio of standard deviations of R&D growth at the 5–year and 1–year horizon, which speaks to the slow adjustment process implied by quadratic adjustment costs, and the kurtosis of R&D growth, which can capture

[31] In the calibration I put additional weight on the first three moments and use absolute differences divided by sums to calculate the error, which is the same error function as in Acemoglu et al. (2018), except for the 6th moment, where I simply use absolute distance as it can be 0 in the data.

the effect of fixed costs of adjustment as they lead to more extreme changes in R&D by making small adjustment unattractive.[32]

Table E.1: Adjustment Costs Model Calibration

| *A. Parameters* | | | |
|---|---|---|---|
| Parameter | Symbol | Value | Calibration |
| Discount factor | $\beta$ | 0.97 | External |
| Intermediate share | $\alpha$ | 0.80 | Terry (2023) |
| R&D scale elasticity | $\gamma$ | 0.50 | Acemoglu et al. (2018) |
| R&D workers | $L_R$ | 0.142 | Acemoglu et al. (2018) |
| Production workers | $L_P$ | 0.858 | Acemoglu et al. (2018) |
| R&D productivity intercept | $\mu$ | -3.329 | Internal |
| R&D productivity dispersion | $\sigma$ | 0.248 | Internal |
| R&D productivity autocorrelation | $\rho$ | 0.889 | Internal |
| Fixed adjustment costs | $\theta^F$ | 0.002 | Internal |
| Quadratic adjustment costs | $\theta^Q$ | 0.578 | Internal |
| *B. Targeted Moments* | | | |
| Moment | Model | Data | Source |
| Growth-rate | 1.50% | 1.50% | Long-run growth rate |
| Std. dev. of R&D productivity growth | 0.402 | 0.401 | Data |
| Autocorrelation of R&D productivity | 0.791 | 0.791 | Data |
| Share \|R&D Growth\| < 2.5% | 12.4% | 12.8% | Data |
| Share \|R&D Growth\| > 25% | 27.4% | 19.8% | Data |
| $\Delta$ Prob. small after large | -4.4% | -4.5% | Data |
| Rel. std. dev. of long- vs short-term R&D Growth | 2.69 | 2.50 | Data |
| Kurtosis of R&D Growth | 2.71 | 2.93 | Data |

*Notes:* Panel A lists parameters and Panel B targeted moments together with their values in the data and the model. See text for a description of the parameters. R&D productivity is measured at the 5-year horizon using the methodology proposed in the main text. Autocorrelation of R&D productivity growth is calculated using the firm-fixed effect robust estimator proposed in Han and Phillips (2010). Kurtosis is calculated excluding the 2.5% lowest and highest observations to safeguard against outliers. See text for details.

The calibration suggests almost no fixed adjustment costs, but features a significant quadratic adjustment cost component. It matches the first three moments almost exactly and provides a decent fit for the remaining moments. The calibration likely features very low fixed adjustment costs as there is a relatively small share of observations with low R&D growth in absolute terms. Larger adjustment costs increase this measure dramatically as shown in

[32] I estimate the change in probability by regressing an indicator for being in the top or bottom quintile in the previous year on an indicator for currently being in the middle two quintiles of R&D growth.

Figure E.6. This sensitivity trumps the desire for higher fixed costs to match the kurtosis of R&D growth. The level of quadratic adjustment costs is likely pinned down by the tug-and-pull of reducing the share of observations with large adjustments against matching the relative dispersion of R&D growth across horizons. As shown in Figure E.7, the former and latter moment suggests larger and smaller quadratic adjustment costs, respectively.

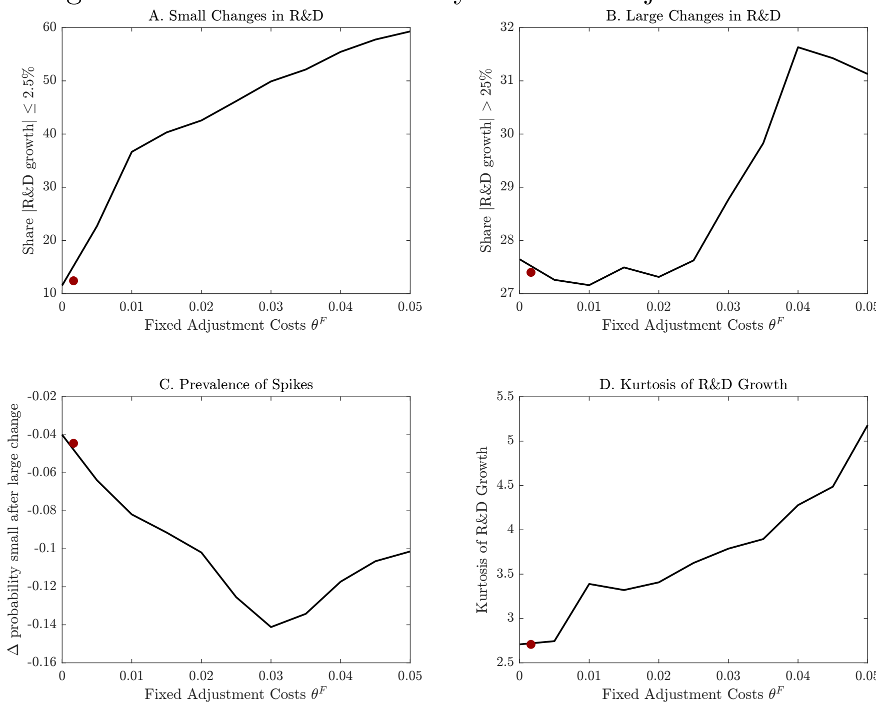
Table E.2: Adjustment Costs Model Counterfactual

| Moment | Data | Model | |
|---|---|---|---|
| | | Main | No adj. cost |
| Growth-rate | 1.50% | 1.50% | 1.52% |
| Std. dev. of R&D return | 0.931 | 0.145 | 0.005 |
| Autocorrelation of R&D return | 0.697 | 0.093 | 0.009 |
| Std. dev. of R&D productivity growth | 0.401 | 0.402 | 0.380 |
| Autocorrelation of R&D productivity | 0.791 | 0.791 | 0.878 |
| Share $\lvert$R&D Growth$\rvert$ $<$ 2.5% | 12.8% | 12.4% | 25.1% |
| Share $\lvert$R&D Growth$\rvert$ $>$ 25% | 19.8% | 27.4% | 74.9% |
| $\Delta$ Prob. small after large | -4.5% | -4.4% | -1.1% |
| Rel. std. dev. of long- vs short-term R&D Growth | 2.50 | 2.69 | 1.93 |
| Kurtosis of R&D Growth | 2.93 | 2.71 | 2.31 |

*Notes:* Table reports data and model moments. The model moments derive from the main calibration and a counterfactual without adjustment costs, i.e., setting $\theta^F = \theta^Q = 0$. The first moment reports the long-term growth rate in the economy. Moments 2 and 3 report the standard deviation and autocorrelation of R&D returns calculated as described in the main text. The remaining moments are the target moments for the calibration. See text for details.

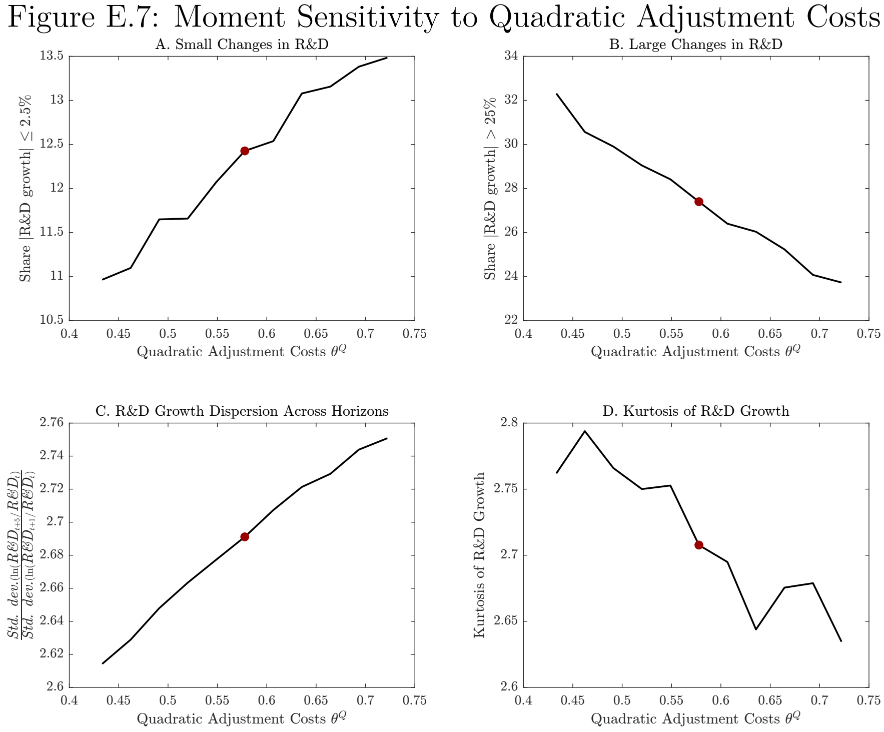
The calibration suggests moderate dispersion in measured R&D returns and slight persistence therein as reported in column (2) of Table E.2. The standard deviation of R&D returns is 0.15, which can account for $\frac{0.15}{0.93} \approx 16\%$ of their dispersion in the data. The persistence of R&D returns, however, is much smaller with a value of 0.1 compared to 0.7 in the data. Thus, the calibrated model can account for some R&D return dispersion, but fails to match their persistence. Unsurprisingly, turning off adjustment costs, as reported in the third column, yields no dispersion in R&D returns and faster growth, as predicted by the theory.

Figure E.6: Moment Sensitivity to Fixed Adjustment Costs




*Notes:* Figure reports model moments for alternative specifications varying fixed adjustment costs $\theta^F$ and keeping all other parameters at their calibrated level. The red dot plots the value at the calibration.

Figure E.7: Moment Sensitivity to Quadratic Adjustment Costs




*Notes:* Figure reports model moments for alternative specifications varying quadratic adjustment costs $\theta^Q$ and keeping all other parameters at their calibrated level. The red dot plots the value at the calibration.

# F Monopsony Power and R&D Wedges

This section investigates the contribution of monopsony power over inventor to R&D return dispersion. I introduce a dynamic heterogeneous firms growth model in line with the framework in the main text and calibrate it with monopsony power over inventors via moment matching. As part of the calibration exercise, I estimate firm-level labor supply elasticities that inform the degree of monopsony power over inventors. Monopsony power in the calibrated model can account for 38% of the empirical R&D return dispersion and can capture most of their persistence in the data.

## F.1 Setup

**Household.** The representative household consists of a unit mass of workers, whereof $L_P$ work in production and the remainder in R&D. Labor supply in R&D is subject to a labor disutility term that depends on R&D labor aggregate $L_{R,t}$. The latter in turn depends on the allocation of R&D workers $\ell_{it}$ across firms. The household owns all firms and receives dividend income $\Pi_t$ in addition to production wages $W_{P,t}$ and firm-specific R&D wages $W_{R,it}$. Income can either be used for consumption $C_t$ or saved in riskless bond $B_t$ paying interest $R_t$. The riskless bond is in zero net-supply. Households have log preferences over consumption and discount the future with factor $\beta$. Their optimization problem is thus given by

$$
\begin{aligned}
U \equiv \max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \cdot \left( \ln C_t - \alpha_R \cdot \frac{\varepsilon}{1+\varepsilon} \cdot \left( \frac{L_{R,t}}{\alpha_R} \right)^{\frac{1+\varepsilon}{\varepsilon}} \right) \\
\text{s.t.} \quad B_{t+1} + C_t = R_t \cdot B_t + \int_0^1 W_{R,it} \cdot \ell_{it} \cdot di + W_{P,t} \cdot L_P + \Pi_t \qquad \text{(F.1)} \\
\text{and} \quad L_{R,t} = \left( \underline{\ell} + \frac{1}{1+\xi} \right)^{-1} \left( \int_0^1 \ell_{kt} \cdot \left( \underline{\ell} + \frac{1}{1+\xi} \left( \frac{\ell_{kt}}{L_{R,t}} \right)^{\xi} \right) dk \right).
\end{aligned}
$$

The labor disutility term for R&D workers is set up such that is collapses to a simple linear aggregator for $\underline{\ell} = \xi = 0$ and to a constant elasticity structure for $\underline{\ell} = 0$. The parameter $\xi$ generally regulated how good of a substitute firms are for each other from the perspective of R&D workers, while $\underline{\ell}$ regulated the degree to which this elasticity of substitution depends on R&D employment.

The optimization problem yields the standard Euler equation

$$
\frac{C_{t+1}}{C_t} = \beta \cdot R_{t+1}. \qquad \text{(F.2)}
$$

The aggregate labor supply for R&D workers satisfies

$$\frac{W_{R,t}}{C_t} = \left(\frac{L_{R,t}}{\alpha_R}\right)^{\frac{1}{\varepsilon}} \quad \text{s.t.} \quad W_{R,t} = \int_0^1 \left(\frac{\ell_{kt}}{L_{R,t}}\right) \cdot W_{R,kt} \cdot dk, \tag{F.3}$$

which confirms that $\varepsilon$ is the aggregate labor supply elasticity. Firm-level labor supply in turn is

$$\frac{W_{R,kt}}{W_{R,t}} = \left(\underline{\ell} + \frac{1}{1+\xi} + \frac{\xi}{1+\xi}\int_0^1 \left(\frac{\ell_{kt}}{L_{R,t}}\right)^{1+\xi} dk\right)^{-1} \left(\underline{\ell} + \left(\frac{\ell_{kt}}{L_{R,t}}\right)^{\xi}\right). \tag{F.4}$$

The associated firm-level labor supply elasticity is then given by

$$\frac{\partial \ln \ell_{kt}}{\partial \ln W_{R,kt}} = \frac{1}{\xi} \cdot \frac{\underline{\ell} + (\ell_{kt}/L_{R,t})^{\xi}}{(\ell_{kt}/L_{R,t})^{\xi}}.$$

Three cases are worth highlighting. First, for $\xi \to 0$, the elasticity converges to $\infty$, i.e., the competitive scenario. Second, for $\bar{\ell} = 0$, the elasticity is the same for all firms and given by $1/\xi$. Finally, larger values of $\bar{\ell}$ will increase the elasticity as long as $\xi > 0$ such that firms act more competitively. However, this effect is more pronounced for firms with low R&D employment such that smaller firms face a more competitive market compared to larger firms, which is in line with the evidence in Berger et al. (2022) and Yeh et al. (2022).

**Production.** The final output in the economy is produced with production labor and intermediate inputs $\{x_{it}\}_{i \in A_t}$:

$$Y_t = L_P^{1-\alpha} \cdot \int_0^{A_t} x_{it}^{\alpha} \cdot di \tag{F.5}$$

Profit maximization yields the demand function for intermediate inputs:

$$p_{it} = \alpha \left(\frac{L_P}{x_{it}}\right)^{1-\alpha}. \tag{F.6}$$

Each intermediate input is owned by one of a unit mass of innovative firms. The firms face constant unit costs of production $\psi$ and set the price as to maximize profits:

$$\pi_{it} \equiv \max \{p_{it} \cdot x_{it} - \psi \cdot x_{it}\} \quad \text{s.t.} \quad (\text{F.6}). \tag{F.7}$$

The profit maximizing price is given by

$$p_{it} = \frac{\psi}{\alpha}. \tag{F.8}$$

I normalize this value to 1 by setting $\psi = \alpha$.

The resulting equilibrium profits are constant across all intermediate inputs and given by

$$\pi_{it} = \pi = (1-\alpha)\alpha^{\frac{1}{1-\alpha}} \cdot L_P. \tag{F.9}$$

Equilibrium output and net-output are given by

$$Y_t = A_t \cdot L_P \cdot \alpha^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad \tilde{Y}_t = Y_t - \psi \int_0^{A_t} x_{it} di = A_t \cdot L_P \cdot \alpha^{\frac{\alpha}{1-\alpha}} \cdot (1-\alpha^2). \tag{F.10}$$

Equilibrium wages for production workers satisfy

$$W_{P,t} = A_t \cdot (1-\alpha) \cdot \alpha^{\frac{\alpha}{1-\alpha}}. \tag{F.11}$$

**Innovation.** There is a unit mass of innovative firms that create new intermediate goods using R&D labor $\ell_{it}$ with idea production function

$$z_{it+1} = A_t \cdot \varphi_{it} \cdot \ell_{it}^{\gamma}. \tag{F.12}$$

R&D productivity $\varphi_{it}$ follows an AR(1) process in logs:

$$\ln \varphi_{it} = (1-\rho) \cdot \mu + \rho \cdot \ln \varphi_{it-1} + \epsilon_{it} \quad \text{with} \quad \epsilon_{it} \sim N(0, \sigma^2). \tag{F.13}$$

The value of invention realized tomorrow is the present discounted value of the associated profits, which is given by

$$\mathcal{V} = R^{-1} \cdot \sum_{t=0}^{\infty} R^{-t} \cdot \pi = \frac{1}{R-1} \cdot \pi. \tag{F.14}$$

R&D workers are hired at wage $W_{R,kt}$ and firms take into account their impact on the wage through their demand for R&D workers, i.e., they anticipate their firm-specific labor supply function.

The firms then choose R&D input $\ell_{it}$ by solving a simple static maximization problem:

$$\max_{\ell_{it}} \{z_{it+1} \cdot \mathcal{V} - W_{R,kt} \cdot \ell_{kt}\} \quad \text{s.t. (F.4) and (F.12).} \tag{F.15}$$

It is straight-forward to show that the first-order conditions for this problem boil down to a simple condition on the R&D return:

$$\frac{z_{it+1} \cdot \mathcal{V}}{W_{R,kt} \cdot \ell_{kt}} = \frac{1}{\gamma}\left(1 + \xi \cdot \frac{(\ell_{kt}/L_{R,t})^{\xi}}{\underline{\ell} + (\ell_{kt}/L_{R,t})^{\xi}}\right).$$

It follows immediately from this formulation that the model can generate R&D return dispersion iff $\bar{\ell} > 0$ and $\xi > 0$, i.e., if the firm-specific labor supply elasticity is finite and different across firm sizes. This formulation can also capture a positive correlation between R&D returns and inventor employment as observed in the data under those conditions. Otherwise, the return is equalized across firms and, thus, independent of R&D employment.

Aggregate profits are given by the profits from production minus R&D costs:

$$\Pi_t = A_t \cdot \pi - \int_0^1 W_{R,kt} \cdot \ell_{kt} \cdot dk \tag{F.16}$$

Finally, the growth rate of the mass of intermediate goods is given by

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \int_0^1 \tilde{z}_{it+1} \cdot di, \tag{F.17}$$

where $\tilde{z}_{it} = z_{it}/A_t$.

**Equilibrium.** Finally, I define a competitive equilibrium and balanced growth path as is customary. Throughout, I assume that the economy is on its long-run balanced growth path.

**Definition 5** (Competitive Equilibrium (CE))**.** *For a given initial level* $\{A_0, \{\varphi_{i0}\}\}$ *and the R&D productivity process* $\{\varphi_{it}\}$ *in* (F.13)*, a CE is a sequence* $\{A_t, C_t, W_{R,t}, R_t, \{\ell_{it}, V_t(\ell_{it-1}, \varphi_{it})\}\}$ *such that households optimize, innovative firms choose R&D employment optimally, and markets clear.*

**Definition 6** (Balanced Growth Path (BGP))**.** *A BGP is a CE such that* $\{A_t, W_{R,kt}(\ell_{kt}), V_t(\cdot)\}$ *grow as a common rate* $g$ *and the interest rate* $R_t$ *is constant.*

I solve the model along the BGP using conventional numerical methods.

## F.2 Labor Market Evidence

The model directly links R&D returns to monopsony power in the R&D labor market, i.e., firms' ability to affect R&D wages via the labor supply elasticity:

$$\frac{z_{it+1} \cdot \mathcal{V}}{W_{R,kt} \cdot \ell_{kt}} = \frac{1}{\gamma}\left(1 + \frac{1}{\epsilon_{it}}\right) \quad \text{with} \quad \epsilon_{it} \equiv \frac{\partial \ln \ell_{R,kt}}{\partial \ln W_{R,kt}} = \frac{1}{\xi} \cdot \frac{\underline{\ell} + (\ell_{kt}/L_{R,t})^{\xi}}{(\ell_{kt}/L_{R,t})^{\xi}}. \tag{F.18}$$

Monopsony power is reflected in R&D returns as firms exploiting it reduce their inventor employment to keep wages low and, thus, have to scale back on R&D, which gives them larger average returns due to lower R&D costs. The extent of this force depends on the firm-specific labor supply elasticity such that firms facing inelastic supply scale back more and, therefore, earn higher returns. Size-dependent heterogeneity in this elasticity, as proposed in the model above, can then account for R&D return dispersion as well as the link between R&D returns and inventor employment.

To calibrate the model credibly, we need direct evidence on the labor supply elasticity in R&D and its link to R&D employment. I fill this gap by estimating that the inventor supply elasticity is indeed smaller for firms with high R&D returns and large R&D employment. This finding directly supports the conclusion that inventor monopsony contributes to R&D return dispersion and the link between R&D employment and returns. I first discuss estimation of the inverse supply elasticity, before linking it to R&D returns and inventor employment.

The inverse labor supply elasticity can be estimated by regressing log changes in the inventor wage on changes in log inventor employment as shown in equation (F.19) (Manning, 2003). The coefficient on the changes in inventor employment identifies the average inverse labor supply elasticity if the error term is uncorrelated with changes in inventor employment.

$$\Delta \ln \text{Inventor Wage}_{it} = \bar{\epsilon} \times \Delta \ln \text{Inventors}_{it} + \alpha_{j(i)\times t} + \varepsilon_{it} \tag{F.19}$$

A natural challenge in this regression are labor supply shocks that simultaneously affect wages and employment. For example, if a firm becomes more attractive to employees for independent reasons, we might expect that the firm will be able to lower wages and hire more workers. However, this variation does not answer the questions as to what happens to wages if the firm wants to expand employment. In other words, supply shocks confound the estimation of a supply elasticity, and we thus need demand shocks for identification.

I follow the approach taken in Seegmiller (2023) closely, by using stock market returns as an instrument for employment, or inventor employment in my case. The idea behind the instrument is that stock market returns reflect changes in firm productivity or demand for a firm's product that incentivize it to expand. A positive demand shock to the firm will not only induce it to expand production, but also increase the potential market size for new products. The latter then gives the firm an incentive to expand R&D as well. The identification assumption is thus that stock market returns do not affect changes in inventor wages other than through their impact on the demand for inventors.

I connect the inverse labor supply elasticity with R&D returns by adding an interaction term for firms with above median R&D return to the regression framework. If R&D return dispersion is partly driven by heterogeneity in the firm-specific labor supply elasticity, then we would expect a positive coefficient on the interaction term, as firms with high R&D returns face a high inverse labor supply elasticity. I follow a similar approach for above and below median inventor employment.

$$\begin{aligned}\Delta \ln \text{Inv. Wage}_{it} = \quad & \epsilon_l \times \Delta \ln \text{Inv.}_{it} \\ & + (\epsilon_h - \epsilon_l) \times \Delta \ln \text{Inv.}_{it} \times \{\text{Above Median R\&D Return}\}_{it} \\ & + \beta \{\text{Above Median R\&D Return}\}_{it} + \alpha_{j(i)\times t} + \varepsilon_{it}\end{aligned} \tag{F.20}$$

The identification assumption for the interaction terms is that the growth rate of R&D wages is not is not linked differentially to stock returns for larger firms other than through their impact on R&D employment.

I do not observe R&D wages directly in the data and, thus, construct a proxy using ratio of R&D spending to inventors at the 5-year level, which is motivated by the high labor share in innovation. According to the NSF BERDS (2019), labor expenditures contribute about 80% of total attributable costs in 2019. Using 5-year windows allows me to pick up medium run effects. Note, however, that the instrument only captures annual variation, which safeguards the estimated coefficient from concerns around the use of long-run averages.

There are several potential challenges to this empirical strategy. First, stock market returns might partly reflect labor supply shock to the degree that they increase firm value. Note, however, that this concern would lead to a downwards bias of the estimated elasticity as supply shocks, such as preference shocks, naturally lower wages and raise employment. This concerns could also bias the interaction coefficient, e.g., the composition of shocks captured by stock returns varies by R&D employment or if the sensitivity of R&D wages to any bias depends on the R&D employment. Second, incentive pay for researchers in stock options could lead to a correlation between returns and inventor wages unrelated to inventor employment.[33] However, there is no potential bias if stock compensation is a fixed share of total labor compensation as changes in the value of stock-based compensation is directly offset by changes in their quantity. I consider the extent of this threat in ongoing work and find it to be small in practice. Note that incentive pay could also bias estimate of the interaction regression, e.g., if firms with larger R&D employment pay out a larger share of salaries in stocks or stock options. Finally, my measure of R&D wages includes non-labor expenditure such that lab equipment, which adds measurement error. Note, however, that such measurement error only biases the regression if it is systematically related to the instrument.

Table F.1: Inventor Inverse Labor Elasticity Estimates

| | (1) | (2) | (3) |
|---|---|---|---|
| | $\Delta \ln$ `Inventor Wage`$_{it}$ | | |
| $\Delta \ln$ `Inventors` | 0.963*** | 0.817** | 0.410** |
| | (0.198) | (0.325) | (0.203) |
| — × {Top 50% R&D Return} | | 1.079** | |
| | | (0.512) | |
| — × {Top 50% Inventors} | | | 1.245*** |
| | | | (0.446) |
| {Top 50% R&D Return} | | -0.224*** | |
| | | (0.044) | |
| {Top 50% Inventors} | | | -0.090*** |
| | | | (0.020) |
| First stage F stat. (Main) | 96 | 39 | 48 |
| First stage F stat. (Inter.) | | 60 | 71 |
| Observations | 14,834 | 14,834 | 14,834 |

*Note:* This reports the second stage results for the main specification. All regressions control for NAICS3 × year fixed effects. Standard errors clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

My estimation results, as reported in Table F.1, reveal three novel findings: first, estimated average inverse labor supply elasticity is significantly different from 0 such that expanding

[33] About 12% of total labor compensation in R&D is stock-related (NSF BERDS 2019).

firms face higher wages.[34] A 1% increase in employment is associated with a 0.96% increase in average wages. The effect size is of comparable magnitude to the estimate of 0.84 for high-skilled workers in Seegmiller (2023), who uses detailed LEHD data on wages and employment. Second, these effects are stronger for firms with high R&D returns. A firm with above median R&D return faces an inverse labor supply elasticity of $0.817 + 1.079 \approx 1.9$ implying that a 1% increase in employment requires a 1.9% increase in wages. Translating the differences in the labor supply elasticity into markdowns, i.e. $1 + 1/\hat{\epsilon}$, I find they can account for around 30% of the average difference in R&D returns between above and below median R&D return firms.[35] Third, column (3) reveals that the inverse labor supply elasticity is indeed larger for firms with high inventor employment, which suggests that the correlation of employment and R&D returns is indeed partly driven by markdowns. Differences in the labor supply elasticity explain approximately the entire difference between R&D returns of above and below median R&D employment firms.[36]

## F.3 Calibration and Counterfactuals

I calibrate the 11 model parameters using a combination of external calibration and moment matching. I calibrate $\beta = 0.97$ to achieve a real interest rate close to 5% in light of a long-run growth-rate of 1.5% and $\alpha = 0.8$ to get a markup of 25% as in Terry (2023). I set the R&D scale elasticity to $\gamma = 0.5$, which implies an R&D unit cost elasticity of -1 in absence of frictions and is standard in the literature (Acemoglu et al., 2018). Following Acemoglu et al. (2018), I assume a production labor force of $L_P = 1 - 0.142 = 0.858$ and will target the R&D labor force such that 14.2% of the overall workforce are employed in R&D. Finally, I set the aggregate labor supply elasticity $\epsilon = 0.5$ in line with the estimates reported in Chetty et al. (2012).

The remaining parameters are calibrated using moment matching as summarized in Table F.2.[37] I target the parameters of the R&D productivity process $\{\mu, \sigma, \rho\}$ with a long-term growth rate of 1.5%, the standard deviation of R&D productivity, and the auto-correlation of R&D productivity corrected for firm-fixed effects as in Han and Phillips (2010). I measure

[34] The first stage results are reported below in Table F.4. The F-statistics for the regression are throughout sufficiently large suggesting that weak instruments are not an issue.

[35] The ratio of average R&D returns above and below the median is 5.7, while the ratio of implied markdowns is $(1 + 1.079 + 0.817)/(1 + 0.817) \approx 1.6$. The ratio of both is $1.6/5.7 \approx 30\%$.

[36] The ratio of average R&D returns for firms with above and below inventor employment is 1.8, while the ratio of implied markdowns is $(1 + 1.245 + 0.410)/(1 + 0.410) \approx 1.9$.

[37] In the calibration I put additional weight on the first four moments and use absolute differences divided by sums to calculate the error, which is the same error function as in Acemoglu et al. (2018).

R&D productivity at the 5-year level as in the main text. As mentioned above, I target an R&D labor force of 14.2%, which determines the R&D labor disutility term $\alpha_R$.[38] Finally, I target the parameters of firm-specific labor supply $\{\underline{\ell}, \xi\}$ by matching the regression evidence in Table F.1. Generally speaking, larger estimated elasticities suggest larger values of $\xi$, while $\underline{\ell}$ controls the degree of size-dependence.

Table F.2: Model Calibration

| *A. Parameters* | | | |
|---|---|---|---|
| Parameter | Symbol | Value | Calibration |
| Discount factor | $\beta$ | 0.97 | External |
| Intermediate share | $\alpha$ | 0.80 | Terry (2023) |
| R&D scale elasticity | $\gamma$ | 0.50 | Acemoglu et al. (2018) |
| Production workers | $L_P$ | 0.858 | Acemoglu et al. (2018) |
| Aggregate R&D supply elasticity | $\epsilon$ | 0.500 | Chetty et al. (2012) |
| R&D productivity intercept | $\mu$ | -3.249 | Internal |
| R&D productivity dispersion | $\sigma$ | 0.304 | Internal |
| R&D productivity autocorrelation | $\rho$ | 0.863 | Internal |
| R&D disutility | $\alpha_R$ | 0.317 | Internal |
| Firm R&D supply elasticity | $\xi$ | 11.25 | Internal |
| Firm R&D supply intercept | $\underline{\ell}$ | 10.82 | Internal |
| *B. Targeted Moments* | | | |
| Moment | Model | Data | Source |
| Growth-rate | 1.50% | 1.50% | Long-run growth rate |
| R&D workers | 0.142 | 0.142 | Acemoglu et al. (2018) |
| Std. dev. of R&D productivity growth | 0.402 | 0.402 | Data |
| Autocorrelation of R&D productivity | 0.791 | 0.791 | Data |
| Average wage elasticity | 0.922 | 0.923 | Table F.1 column (1) |
| Wage elasticity for low R&D return | 0.505 | 0.817 | Table F.1 column (2) |
| $\Delta$ wage elasticity for high R&D return | 1.138 | 1.079 | Table F.1 column (2) |
| Wage elasticity for small employers | 0.533 | 0.410 | Table F.1 column (3) |
| $\Delta$ wage elasticity for large employers | 1.176 | 1.245 | Table F.1 column (3) |

*Notes:* Panel A lists parameters and Panel B targeted moments together with their values in the data and the model. See text for a description of the parameters. R&D productivity is measured at the 5-year horizon using the methodology proposed in the main text. Autocorrelation of R&D productivity growth is calculated using the firm-fixed effect robust estimator proposed in Han and Phillips (2010). See text for details.

The calibration suggests a significant role for both parameters of the R&D disutility function. It matches the first five moments almost exactly and provides a decent fit for the remaining

[38] I target the realized R&D workforce $\tilde{L}_R = \int_0^1 \ell_{kt} dk$ here rather than $L_R$, which I interpret as a disutility term measuring how costly the provision of research labor is.

moments. Importantly, it features a slightly too low wage elasticity for firms with low R&D returns as well as for the combined R&D elasticity for high R&D return firms. On the other hand, it features a slightly too high wage elasticity for small employer and a slightly too high combined elasticity for large employers. Elasticities for low R&D return and employment firms are very similar as are the gaps to high R&D return and employment firms. This feature follows immediately from the fact that R&D employment is the only driver of differences in R&D return in the model such that R&D returns and employment are perfectly correlated. The excessive steepness for the specification differentiating firms by R&D is thus less concerning to the degree that there are other drivers of R&D returns in the data that are uncorrelated with R&D employment.

Table F.3: Adjustment Costs Model Counterfactual

| Moment | Data | Model | | |
|---|---|---|---|---|
| | | Main | No monopsony | |
| | | | Fixed $\tilde{L}_R$ | Full |
| Growth-rate | 1.50% | 1.50% | 1.56% | 1.84% |
| R&D employment | 0.142 | 0.142 | 0.142 | 0.198 |
| Std. dev. of R&D return | 0.931 | 0.357 | 0.005 | 0.005 |
| Autocorrelation of R&D return | 0.697 | 0.586 | 0.004 | 0.005 |
| Std. dev. of R&D productivity growth | 0.402 | 0.402 | 0.301 | 0.301 |
| Autocorrelation of R&D productivity | 0.791 | 0.791 | 0.768 | 0.768 |
| Average wage elasticity | 0.923 | 0.922 | 0.982 | 0.978 |
| Wage elasticity for low R&D return | 0.817 | 0.505 | 0.957 | 0.979 |
| $\Delta$ wage elasticity for high R&D return | 1.079 | 1.138 | 0.050 | -0.003 |
| Wage elasticity for small employers | 0.410 | 0.533 | 0.499 | 0.498 |
| $\Delta$ wage elasticity for large employers | 1.245 | 1.176 | 2.417 | 2.408 |

*Notes:* Table reports data and model moments. The model moments derive from the main calibration and a counterfactual without monopsony, i.e., when the firm ignores the R&D supply elasticity. The first moment reports the long-term growth rate in the economy. Moments 3 and 4 report the standard deviation and autocorrelation of R&D returns calculated as described in the main text. The remaining moments are the target moments for the calibration. See text for details.

The calibration suggests significant dispersion in measured R&D returns and strong persistence therein as reported in column (2) of Table F.3. The standard deviation of R&D returns is 0.36, which can account for $\frac{0.36}{0.93} \approx 38\%$ of their dispersion in the data. The persistence of R&D returns is slightly smaller with a value of 0.6 compared to 0.7 in the data. Thus, the calibrated model can account for a significant fraction of R&D return dispersion as well as their persistence. Accounting for persistence naturally arises as R&D returns are directly linked to R&D productivity in this context, which itself is very persistent. Unsurprisingly, turning off monopsony power by forcing firm to act as price-takers yields no dispersion in R&D returns and faster growth, as predicted by the theory. Column (3) reports counterfactual values holding constant total R&D employment, while R&D employment at the aggregate level is allowed to adjust in column (4). Importantly, if firms act as price takers, then there are no differences in R&D returns across firms and, thus, also no difference in the wage elasticity across firms along the R&D return spectrum as shown in the ninth row. Finding a difference across firms with high and low R&D returns, thus, suggests that these firms do not act as price takers. Finally, the growth acceleration in absence of monopsony power is significant. Holding constant R&D employment, I find that growth accelerates by 0.06 pp, while if accelerates by 0.24 pp if R&D employment is permitted to adjust as well.

## F.4 First Stage Results

Table F.4 reports the first-stage results for the main specification.

Table F.4: Inventor Inverse Labor Elasticity Estimates — First Stage

| | (1) | (2) | (3) |
|---|---|---|---|
| **A. Main** | | $\Delta \ln \texttt{Inventors}_{it}$ | |
| Stock Return$_{it}$ | 0.065*** | 0.042*** | 0.065*** |
| | (0.007) | (0.009) | (0.010) |
| — × {Top 50% R&D Return} | | 0.042*** | |
| | | (0.011) | |
| — × {Top 50% Inventors} | | | 0.001 |
| | | | (0.011) |
| **B. Interaction** | $\Delta \ln \texttt{Inventors}_{it}$ × **{Top 50% R&D Return$_{it}$}** | | |
| Stock Return$_{it}$ | | 0.002 | 0.007** |
| | | (0.002) | (0.003) |
| — × {Top 50% R&D Return} | | 0.047*** | |
| | | (0.007) | |
| — × {Top 50% Inventors} | | | 0.039*** |
| | | | (0.008) |
| First stage F stat. (Main) | | 39 | 48 |
| First stage F stat. (Inter.) | | 39 | 48 |
| Observations | 14,834 | 14,834 | 14,834 |

*Note:* First stage regression results for main specification. All regressions control for NAICS3 × year fixed effects. Standard errors clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

# Online Appendix

*Not for publication*

## G Mechanisms Driving R&D Wedges

In this section I briefly highlight some potential drivers in R&D returns and impact-value factors. For this purpose, I will rely on a two-period growth model.

### G.1 Baseline Model

**Setup.** The final good producer creates consumption good $Y_t$ by combining inputs $y_{jt}$ from a unit mass of product lines according to:

$$\ln Y_t = \int_0^1 \ln y_{jt} \cdot dj.$$

Each input is supplied by a single monopolist with constant marginal $\psi/A_{jt}$. The monopolist is free to chose any price $p_{jt}$, however, there is a competitive fringe of firm with constant unit costs $\lambda_{jt} \cdot (\psi/A_{jt})$ that limit the monopolists' price setting power. Consequently, the monopolist sets limit price equal to the marginal costs of the competitive fringe and earns profits

$$\pi_{jt} = Y_t \cdot (1 - \lambda_{jt}^{-1}).$$

There is a unit mass of innovative firms at time 0, which may hire inventors $\ell_i$ at wage $W$ to produce an invention at time 1 with probability $z_i$:

$$z_i = \varphi_i \cdot \ell_i^{\gamma}.$$

An invention improves technology in a random product line by $\lambda_i$ such that $A_{j1} = \lambda_i \cdot A_{j0}$ in a product line with a successful invention. The competitive fringe then absorbs the knowledge of the previous monopolist, such that its unit cost gap to the monopolist is $\lambda_i$ as well. Resultingly, the innovation yields profits $\pi_i$ in period 1, which firms discount at rate $R$. The value of innovation to the firm is thus given by $V_i = \pi_i/R$ and its optimization problem

$$\max_{\ell_i} \{V_i \cdot z_i - W \cdot \ell_i\}$$

There is a fixed number of research workers, whose labor market clearing condition determines the R&D wage in equilibrium:

$$L = \int_0^1 \ell_i \cdot di.$$

Finally, I define the productivity index $A_t$ such that $\ln A_t = \int_0^1 \ln A_{jt} \cdot dj$. Consequently, its growth rate is given by

$$g = \ln(A_1/A_0) \approx \int_0^1 (\lambda_i - 1) \cdot z_i \cdot di,$$

where the approximation relies on $\ln \lambda_i \approx \lambda_i - 1$.

The planner maximizes economic growth subject to the same technological constraints as firms:

$$g^* = \max \int_0^1 z_i \cdot (\lambda_i - 1) \cdot di \quad \text{s.t.} \quad L = \int_0^1 \ell_i \cdot di.$$

**R&D returns and impact-value factors.** It is straight-forward to show that in this setup R&D returns are equalized across firms:

$$\frac{V_i \cdot z_i}{W \cdot \ell_i} = \frac{1}{\gamma} \qquad \text{and} \quad \ell_i = \left( \frac{V_i \cdot \varphi_i}{(W/\gamma)} \right)^{\frac{1}{1-\gamma}}.$$

Furthermore, one can show that this allocation is also the solution to

$$g = \max \int_0^1 z_i \cdot V_i \cdot di \quad \text{s.t.} \quad L = \int_0^1 \ell_i \cdot di.$$

Defining $\zeta_i \equiv (\lambda_i - 1)/V_i$, we can thus rearrange the planner problem as

$$g^* = \max \int_0^1 z_i \cdot V_i \cdot \zeta_i \cdot di \quad \text{s.t.} \quad L = \int_0^1 \ell_i \cdot di. \tag{G.1}$$

From the formulation of $V_i$ it then follows immediately that planner and private allocation coincide iff $\zeta_i$ is a constant across firms.

## G.2 Mechanisms for R&D Return Dispersion

**R&D Subsidies or Taxes.** Suppose firms face R&D subsidies $\tau_i$ on their gross R&D expenditure. The firm problem is then given by

$$\max_{\ell_i} \{V_i \cdot z_i - (1-\tau_i) \cdot W \cdot \ell_i\}.$$

Consequently, firms' R&D returns directly reflect differences in subsidy rates:

$$\frac{V_i \cdot z_i}{W \cdot \ell_i} = \frac{1}{\gamma} \cdot (1-\tau_i) \qquad \text{and} \quad \ell_i = \left(\frac{V_i \cdot \varphi_i}{(W/\gamma) \cdot (1-\tau_i)}\right)^{\frac{1}{1-\gamma}}.$$

**Capacity constraints.** Suppose firms face exogenous capacity constraint $\ell_i \leq \bar{\ell}_i$. The firm problem is then given by

$$\max_{\ell_i} \left\{V_i \cdot z_i - W \cdot \ell_i \quad \text{s.t.} \quad \ell_i \leq \bar{\ell}_i\right\}.$$

Consequently, firms' R&D returns directly reflect the tightness of the capacity constraint $\tilde{\lambda}_i$:

$$\frac{V_i \cdot z_i}{W \cdot \ell_i} = \frac{1}{\gamma} \cdot (1+\tilde{\lambda}_i) \qquad \text{and} \quad \ell_i = \left(\frac{V_i \cdot \varphi_i}{(W/\gamma) \cdot (1+\tilde{\lambda}_i)}\right)^{\frac{1}{1-\gamma}}.$$

**Heterogeneous Discount Rates.** Suppose firms have heterogeneous discount rates $R_i$ reflecting e.g. risk or financial constraints, which are not observed in the data. Let $V_i = \pi_i/R$ with $R = \mathbb{E}[R_i]$, then the firm problem is given by

$$\max_{\ell_i} \{(R/R_i) \cdot V_i \cdot z_i - W \cdot \ell_i\}.$$

Consequently, firms' measured R&D returns directly reflect differences in discount factor:

$$\frac{V_i \cdot z_i}{W \cdot \ell_i} = \frac{1}{\gamma} \cdot \frac{R_i}{R} \qquad \text{and} \quad \ell_i = \left(\frac{V_i \cdot (R/R_i) \cdot \varphi_i}{(W/\gamma)}\right)^{\frac{1}{1-\gamma}}.$$

**Adjustment costs.** Suppose firms face exogenous adjustment costs $\phi \cdot W \cdot (\ell_i - \bar{\ell}_i)^2$. The firm problem is then given by

$$\max_{\ell_i} \left\{V_i \cdot z_i - W \cdot \ell_i - \phi \cdot W \cdot (\ell_i - \bar{\ell}_i)^2\right\}.$$

Consequently, firms' R&D returns directly reflect the adjustment costs:

$$\frac{V_i \cdot z_i}{W \cdot \ell_i} = \frac{1}{\gamma} \cdot (1 + 2 \cdot \phi \cdot (\ell_i - \bar{\ell}_i)) \quad \text{and} \quad \frac{V_i \cdot z_i}{W \cdot \ell_i + \phi \cdot W \cdot (\ell_i - \bar{\ell}_i)^2} = \frac{1}{\gamma} \cdot \frac{1 + 2 \cdot \phi \cdot (\ell_i - \bar{\ell}_i)}{1 + \phi \frac{(\ell_i - \bar{\ell}_i)^2}{\ell_i}}$$

Firms with high R&D employment vis-à-vis their reference point have higher R&D returns and vice versa.

**Monopsony Power.** Suppose R&D labor is partly specialized across fields. R&D labor is perfectly mobile across firms within a field, but not across fields, such that the labor market clearing condition is given by

$$L = \int_0^1 \ell_i \cdot \left( \frac{\frac{1}{N_i} \sum_{i \in \mathcal{N}_i} \ell_j}{L} \right)^{\xi} \cdot di, \tag{G.2}$$

where $N_i$ is the number of firms in a given field.

Resultingly, wages may differ across fields and are generally increasing in the average demand for R&D input within a given field:

$$W_i = W \cdot \left( \frac{\frac{1}{N_i} \sum_{j \in \mathcal{N}_i} \ell_j}{L} \right)^{\xi} \tag{G.3}$$

Firm internalize the impact labor demand on wages and, consequently, their first order conditions conditions under symmetry ($\ell_j = \ell_i$ for $j \in \mathcal{N}_i$) are given by

$$\gamma \cdot \theta \cdot \ell_i^{\gamma - 1} = \left( 1 + \frac{1}{N_i} \cdot \xi \right) \cdot W_i \tag{G.4}$$

R&D return is given by $(1/\gamma) \cdot \left(1 + \frac{1}{N_i} \cdot \xi\right)$ with $\Delta_i = \frac{1}{N_i} \cdot \xi$. Variation in R&D returns is thus directly linked to the degree of competition in the firm-specific labor market. Firms with more competition for R&D workers have lower R&D returns and vice versa.

## G.3 Mechanisms for Dispersion in Impact-Value Factors

**Patent Protection.** Suppose that the competitive fringe learns with probability $1 - P_i$ about the new technology of a monopolist such that the monopolist is only able to profit from the innovation with probability $P_i$. In this case, the private value of the invention is

$V_i = P_i \cdot \pi_i / R$, while the public value remains $\lambda_i - 1$. Resultingly, variation in $P_i$ induces variation in $\zeta_i$. Note, also, the that private return is still equalized in this scenario.

**Exogenous Markup Differences.** Suppose that firms differ in their unit cost parameter $\psi_i$ due to e.g. technological differences or complementarities across product lines. The profit of an invention is then given by $\pi_i = Y_1 \cdot (1 - (\psi/\psi_i) \cdot \lambda^{-1})$. Resultingly, variation in $\psi_i$ across firms yields variation in the private value a firm creates from innovation without changing the growth impact $\lambda_i - 1$, which induces variation in impact-value factor $\zeta_i$.

**Endogenous Markup Differences.** Suppose that firms differ in their step-size $\lambda_i$, then $\zeta_i \propto \lambda_i$ such that variation in step-sizes yields variation in impact-value factor. Intuitively, the growth gains of $\lambda_i$ are linear, while the profit gains are concave, such that firms with high quality innovation under-invest in R&D.

**Frictions in the Product Market.** It is straight-forward to see that any frictions in the product market that affect $\pi_i$ without changing the growth-impact of an invention will naturally yield variation in $\zeta_i$ as well. Firms with artificially low profits will under-provide innovation.

**Knowledge externalities.** More general knowledge externalities can also variation in the impact-value factor. For example, let the growth rate be

$$g = \left( \int_0^1 \varphi_i \cdot z_i \cdot (\zeta_i \cdot V_i) \cdot di \right)^{\phi} \cdot \int_0^1 z_i \cdot (\zeta_i \cdot V_i) \cdot di, \tag{G.5}$$

where the first term on the right-hand size captures simultaneous knowledge externalities. Here, the marginal benefit to R&D as perceived by the firm for high $\varphi_i$ firms will be generally too low compared to the social planner perspective if $\phi > 0$ and vice versa.

# H Exploring Measured Impact-Value Factors

This section provides stylized facts on impact-value factors and explore underlying mechanisms—as highlighted in Appendix Section G—empirically. Impact-value factors, as defined in Section 2, capture the alignment of growth impact and private valuation of innovation. I measure them as the ratio of patent forward-citations, which is accurate if citations measure the growth-impact up to a constant factor, while valuations measure private value created. The former is broadly in line with the literature (Akcigit and Kerr, 2018). Ayerst (2022) interprets the same ratio similarly, but focuses on knowledge externalities.

As reported in Appendix Table H.1, measured impact-value factors are highly dispersed across firms, even within narrow industry×year cells. Similar to R&D returns, dispersion in impact-value factors has been rising over time. Furthermore, dispersion is a robust finding across alternative measures and differences in impact-value factor are highly persistent over time. Note, however, that differences in impact-value factor across firms are not as surprising from a theoretical perspective as they are not necessarily linked to optimizing behavior.

Table H.1: Dispersion and Persistence in Impact-Value Factors

| Within Cell | Standard deviation | 5-Year autocorrelation | Observations |
|---|---|---|---|
| *A. Across Industries* | | | |
| — | 1.38 | 0.851*** (0.006) | 11,845 |
| Year | 1.28 | 0.899*** (0.006) | 11,845 |
| **NAICS3 × Year** | 1.11 | 0.858*** (0.007) | 11,845 |
| NAICS6 × Year | 0.99 | 0.831*** (0.008) | 9,764 |
| *B. Across Time* | | | |
| **1975–2014** | 1.11 | 0.856*** (0.007) | 11,845 |
| 1975–1990 | 0.86 | 0.915*** (0.012) | 3,340 |
| 2000–2014 | 1.21 | 0.870*** (0.011) | 5,469 |
| *C. Across Measures* | | | |
| Citations/Valuations | 1.11 | 0.858*** (0.007) | 11,845 |
| Text-Impact/Valuations | 1.01 | 0.885*** (0.009) | 7,560 |
| **Citations/Δ Sales** | 1.39 | 0.665*** (0.009) | 11,688 |
| Markup | 0.18 | 0.480*** (0.009) | 10,615 |
| Profit-implied Markup | 0.31 | 0.565*** (0.008) | 11,845 |

*Note:* Table reports dispersion and autocorrelation of impact-value factors. Unless noted otherwise, the impact-value factor is measured as citations divided by valuations in logs and residualized with respect to NAICS3×Year fixed-effects. Measure in bold is used in the main text. Panel A residualizes with respect to alternative fixed-effects, Panel B uses alternative samples, and Panel C uses alternative measures. See text and Appendix for additional details.

I report empirical evidence regarding the mechanisms driving differences in impact-value factors in Table H.2. First, I find in Panel A that measures of profitability and markups are negatively correlated with the impact-value factor and have significant explanatory power. For example, a one percentage points larger profit rate is associated with a 1.1% lower impact-value factor and the associated $R^2$ is 12%. This finding is in line with de Ridder (2023) and Aghion et al. (2023), who propose theories with partly exogenous markup differences across firms. High markups lead firms to over-invest in R&D vis-à-vis a growth-maximizing allocation and, thus, give them low impact-value factors. The finding is also in line with a world in which quality differences in innovation are only partly reflected in profit rates, which naturally occurs in limit pricing setups.

Second, I find in Panel B that larger and older firms tend to have lower impact-value factors. One potential interpretation is that these firms are better able to capture the underlying value of innovations, giving them lower impact-value factors vis-à-vis young and small firms. For example, Acemoglu et al. (2018) argue that old and less innovative firms might have excessive demand for R&D resources due to overhead costs, which would also give then lower impact-value factors. Relatedly, Abrams et al. (2018), Mezzanotti (2021) and Manera (2022) argue that firms might use the patent system strategically to limit competitor innovation. Inspired by Abrams et al. (2018), I interpret the share of patents receiving exactly 0 citations as a measure of defensive patenting and find in Panel C that it is negatively correlated with the impact value wedge, as expected.

Third, I investigate measures of private frictions in Panel D. König et al. (2022) show that constraints in the product market directly impact private incentives to innovate, giving more constrained firms higher impact-value factors. In contrast, I find no significant correlation with the return on capital and a negative correlation with Tobin's Q. On the other hand, I find that knowledge-intensive firms, which Ewens et al. (2022) argue might be particularly financially constrained, do have higher impact-value factors. Note, however, that mixed findings for measures of frictions might be expected at this point, given the limited evidence for their empirical relevance for R&D returns.

Finally, as discussed in Section 2, optimal R&D policy suggests that firms with larger impact-value factors should receive higher subsidies. In line with with prescription, I find in Panel E that R&D user costs tend to be negatively associated with the impact-value factor and that investment tax credits have a positive correlation. Note, however, that their explanatory power is low.

Table H.2: Correlations with the Impact-Value Factor

| Variable | Estimate | Std. err. | $R^2$ | Observations |
|---|---|---|---|---|
| *A. Profitability* | | | | |
| Profit rate | -1.130*** | (0.112) | 12.0% | 12,044 |
| Markups | -0.486** | (0.193) | 2.5% | 10,775 |
| Alt. markups | -1.623*** | (0.203) | 7.8% | 10,775 |
| *B. Firm Size & Age* | | | | |
| Employment | -0.409*** | (0.030) | 32.5% | 11,995 |
| R&D expenditure | -0.422*** | (0.045) | 28.4% | 11,845 |
| Inventors | -0.377*** | (0.053) | 13.5% | 12,072 |
| Firm age | -0.026*** | (0.003) | 10.3% | 12,072 |
| *C. Defensive Patenting* | | | | |
| 0-cite patents | -4.708*** | (0.331) | 18.0% | 12,072 |
| *D. Frictions* | | | | |
| Return on capital | 0.032 | (0.069) | 0.0% | 12,018 |
| Tobin's Q | -0.091*** | (0.026) | 0.9% | 10,636 |
| Knowledge-intensity | 0.233*** | (0.059) | 2.4% | 11,973 |
| *E. Taxation* | | | | |
| R&D user cost $1 - \tau$ | -1.598* | (0.966) | 0.4% | 11,451 |
| Alt. R&D user cost $1 - \tau$ | -1.192 | (1.070) | 0.2% | 11,691 |
| Public patent involvement | -0.590 | (1.441) | 0.0% | 12,072 |
| Investment tax credits | 0.059*** | (0.015) | 0.9% | 11,381 |

*Note:* Each row reports the regression coefficient of a separate regression with dependent variable log impact-value factor, calculated as the ratio of patent citations to valuations. All regressions control for NAICS3×Year fixed effects and standard errors are clustered at the NAICS6 level. The profit rate is the ratio of profits to sales in level. Markups are $\mu_0$ and $\mu_2$ from Loecker et al. (2020) in logs. Employment, R&D expenditure, and inventors are in logs. 0-Cite Patents is the fraction of patents with no citation in logs. Return on capital is log of the ratio of sales to last periods capital stock. Tobin's Q is the ratio of market valuation to book value in logs. Knowledge intensity is the log of theratio of knowledge capital from Ewens et al. (2022) to the sum of physical and knowledge capital. R&D user cost are from Lucking (2019) and mapped from the state-level to the firm either via the patent location or headquarters. Investment tax credits is the lof of the ratio of investment tax to investment.

# I Top 50 and Bottom 50 Firms by Return on R&D

Table I.1: Top and Bottom R&D Wedge Companies

| Rank | Company Name | Average ln R&D Wedge |
|---|---|---|
| 1 | BJ SERVICES CO | 3.88 |
| 2 | INTUITIVE SURGICAL INC | 3.76 |
| 3 | AT&T INC | 3.68 |
| 4 | CAMERON INTERNATIONAL CORP | 3.55 |
| 5 | ILLINOIS TOOL WORKS | 3.45 |
| 6 | SALESFORCE.COM INC | 3.42 |
| 7 | WEATHERFORD INTL PLC | 3.38 |
| 8 | CREE INC | 3.33 |
| 9 | ARCHER-DANIELS-MIDLAND CO | 3.33 |
| 10 | INTL PAPER CO | 3.32 |
| 11 | MOBIL CORP | 3.22 |
| 12 | HALLIBURTON CO | 3.22 |
| 13 | UNOCAL CORP | 3.20 |
| 14 | DELL TECHNOLOGIES INC | 3.19 |
| 15 | CONOCOPHILLIPS | 3.18 |
| 16 | EXXON MOBIL CORP | 3.16 |
| 17 | ALIGN TECHNOLOGY INC | 3.16 |
| 18 | DEXCOM INC | 3.14 |
| 19 | BAKER HUGHES INC | 3.14 |
| 20 | QUALCOMM INC | 3.13 |
| 21 | OCCIDENTAL PETROLEUM CORP | 2.99 |
| 22 | ALZA CORP | 2.98 |
| 23 | TEXACO INC | 2.96 |
| 24 | ATLANTIC RICHFIELD CO | 2.95 |
| 25 | CHEVRON CORP | 2.95 |

*Note:* This table reports the company names of firms with the best and worst average ln R&D wedge in the sample. I restrict the list to firms with at least 10 observations. See Section 3 for details on the data construction.

Table I.2: Top and Bottom R&D Wedge Companies (continued)

| Rank | Company Name | Average ln R&D Wedge |
|---|---|---|
| 26 | BLACKBERRY LTD | 2.95 |
| 27 | AMOCO CORP | 2.95 |
| 28 | LINDSAY CORP | 2.95 |
| 29 | RED HAT INC | 2.94 |
| 30 | U S SURGICAL CORP | 2.94 |
| 31 | RESMED INC | 2.91 |
| 32 | AKAMAI TECHNOLOGIES INC | 2.89 |
| 33 | STANDARD OIL CO | 2.89 |
| 34 | ALTERA CORP | 2.88 |
| 35 | MICRON TECHNOLOGY INC | 2.87 |
| 36 | UNIVERSAL DISPLAY CORP | 2.87 |
| 37 | SUNPOWER CORP | 2.85 |
| 38 | FORTINET INC | 2.84 |
| 39 | SYMBOL TECHNOLOGIES | 2.83 |
| 40 | BEAM INC | 2.83 |
| 41 | ECOLAB INC | 2.81 |
| 42 | BROADCOM INC | 2.81 |
| 43 | COOPER INDUSTRIES PLC | 2.79 |
| 44 | ALPHABET INC | 2.79 |
| 45 | SANDISK CORP | 2.77 |
| 46 | WEST PHARMACEUTICAL SVSC INC | 2.74 |
| 47 | APPLE INC | 2.74 |
| 48 | ACUITY BRANDS INC | 2.73 |
| 49 | DIGIMARC CORP | 2.73 |
| 50 | KERR-MCGEE CORP | 2.72 |

*Note:* This table reports the company names of firms with the best and worst average ln R&D wedge in the sample. I restrict the list to firms with at least 10 observations. See Section 3 for details on the data construction.

Table I.3: Top and Bottom R&D Wedge Companies (continued)

| Rank | Company Name | Average ln R&D Wedge |
|---|---|---|
| 419 | AEROQUIP-VICKERS INC | 0.67 |
| 420 | AEROJET ROCKETDYNE HOLDINGS | 0.67 |
| 421 | SILICON GRAPHICS INC | 0.66 |
| 422 | AMERICAN AXLE & MFG HOLDINGS | 0.65 |
| 423 | COHERENT INC | 0.65 |
| 424 | AVID TECHNOLOGY INC | 0.65 |
| 425 | ITRON INC | 0.65 |
| 426 | TELLABS INC | 0.64 |
| 427 | GOULD INC | 0.63 |
| 428 | MILACRON INC | 0.60 |
| 429 | RIGEL PHARMACEUTICALS INC | 0.57 |
| 430 | BECKMAN COULTER INC | 0.55 |
| 431 | MAXYGEN INC | 0.55 |
| 432 | HASBRO INC | 0.54 |
| 433 | MICROVISION INC | 0.53 |
| 434 | FIRESTONE TIRE & RUBBER CO | 0.52 |
| 435 | APPLIED MICRO CIRCUITS CORP | 0.51 |
| 436 | MODINE MANUFACTURING CO | 0.48 |
| 437 | FORD MOTOR CO | 0.46 |
| 438 | DIGITAL EQUIPMENT | 0.46 |
| 439 | CELANESE CORP-OLD | 0.46 |
| 440 | SCOTT TECHNOLOGIES INC | 0.45 |
| 441 | AXCELIS TECHNOLOGIES INC | 0.42 |
| 442 | ANALOGIC CORP | 0.41 |
| 443 | QUANTUM CORP | 0.40 |

*Note:* This table reports the company names of firms with the best and worst average ln R&D wedge in the sample. I restrict the list to firms with at least 10 observations. See Section 3 for details on the data construction.

Table I.4: Top and Bottom R&D Wedge Companies (continued)

| Rank | Company Name | Average ln R&D Wedge |
|---|---|---|
| 444 | AMDOCS | 0.39 |
| 445 | DATA GENERAL CORP | 0.39 |
| 446 | SPERRY CORP | 0.37 |
| 447 | ELECTRO SCIENTIFIC INDS INC | 0.35 |
| 448 | NAVISTAR INTERNATIONAL CORP | 0.31 |
| 449 | MAXTOR CORP | 0.30 |
| 450 | QLOGIC CORP | 0.29 |
| 451 | MCDONNELL DOUGLAS CORP | 0.29 |
| 452 | TANDEM COMPUTERS INC | 0.29 |
| 453 | TELECOMMUNICATION SYS INC | 0.24 |
| 454 | ROBINS (A.H.) CO | 0.23 |
| 455 | SPANSION INC | 0.17 |
| 456 | ELECTRONICS FOR IMAGING INC | 0.15 |
| 457 | EXTREME NETWORKS INC | 0.13 |
| 458 | WANG LABS INC | 0.09 |
| 459 | BIO-RAD LABORATORIES INC | 0.08 |
| 460 | DAY INTERNATIONAL INC | 0.08 |
| 461 | ROGERS CORP | -0.08 |
| 462 | SMITH (A.O.) | -0.09 |
| 463 | GENERAL MOTORS CO | -0.14 |
| 464 | AMDAHL CORP | -0.30 |
| 465 | VISTEON CORP | -0.34 |
| 466 | MENTOR GRAPHICS CORP | -0.35 |
| 467 | DE SOTO INC | -0.37 |
| 468 | DONNELLY CORP | -0.40 |

*Note:* This table reports the company names of firms with the best and worst average ln R&D wedge in the sample. I restrict the list to firms with at least 10 observations. See Section 3 for details on the data construction.

# J Second-Order Approximation

*Proof of Lemma 1.* The formula for R&D efficiency is given by

$$\Xi_t = \frac{\int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{\gamma}{1-\gamma}} \cdot di}{\left(\int_0^1 \omega_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}} \cdot di\right)^{\gamma}}, \tag{J.1}$$

where I defined $\tilde{\zeta}_{it} = \int_0^1 \omega_{it} \cdot \zeta_{it} \cdot di$ and $\delta_{it} = (1 + \Delta_{it})$.

Thus, we have

$$\ln \Xi_t = \underbrace{\ln \int_0^1 \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{\gamma}{1-\gamma}} \cdot di}_{\equiv \ln A_t} - \gamma \cdot \underbrace{\int_0^1 \omega_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}} \cdot di}_{\equiv \ln B_t}.$$

Furthermore, in a symmetric steady state with $\tilde{\zeta}_{it} = 1$ and $\delta_{it} = 1$, we have $A = B = 1$. Note that that $\delta_{it} = 1$ is without loss of generality in this case as the formula is HD(0) in a common factor for $\delta$.

The derivatives of the system with respect to the respective elements are then given by

$$
\begin{aligned}
\frac{\partial \ln A_t}{\partial \tilde{\zeta}_{it}} &= \frac{1}{A_t} \cdot \omega_{it} \cdot \delta_{it}^{-\frac{\gamma}{1-\gamma}} \\
\frac{\partial \ln A_t}{\partial \delta_{it}} &= -\frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t} \cdot \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}} \\
\frac{\partial^2 \ln A_t}{(\partial \tilde{\zeta}_{it})^2} &= -\left(\frac{1}{A_t} \cdot \omega_{it} \cdot \delta_{it}^{-\frac{\gamma}{1-\gamma}}\right)^2 \\
\frac{\partial^2 \ln A_t}{(\partial \delta_{it})^2} &= -\left(\frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t} \cdot \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}}\right)^2 + \frac{\gamma}{(1-\gamma)^2} \cdot \frac{1}{A_t} \cdot \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{2-\gamma}{1-\gamma}} \\
\frac{\partial^2 \ln A_t}{\partial \delta_{it} \cdot \partial \delta_{jt}} &= -\left(\frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t} \cdot \omega_{it} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}}\right)\left(\frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t} \cdot \omega_{jt} \cdot \tilde{\zeta}_{jt} \cdot \delta_{jt}^{-\frac{1}{1-\gamma}}\right) \\
\frac{\partial^2 \ln A_t}{\partial \tilde{\zeta}_{it} \cdot \partial \delta_{it}} &= \frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t^2} \cdot \omega_{it}^2 \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{1+\gamma}{1-\gamma}} - \frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t} \cdot \omega_{it} \cdot \delta_{it}^{-\frac{1}{1-\gamma}} \\
\frac{\partial^2 \ln A_t}{\partial \tilde{\zeta}_{jt} \cdot \partial \delta_{it}} &= \frac{\gamma}{1-\gamma} \cdot \frac{1}{A_t^2} \cdot \omega_{it} \cdot \omega_{jt} \cdot \tilde{\zeta}_{it} \cdot \delta_{it}^{-\frac{\gamma}{1-\gamma}} \cdot \delta_{jt}^{-\frac{1}{1-\gamma}} \\
\frac{\partial(\gamma \cdot \ln B_t)}{\partial \delta_{it}} &= -\frac{\gamma}{1-\gamma} \cdot \frac{1}{B_t} \cdot \omega_{it} \cdot \delta_{it}^{-\frac{2-\gamma}{1-\gamma}} \\
\frac{\partial^2(\gamma \cdot \ln B_t)}{(\partial \delta_{it})^2} &= -\frac{\gamma}{(1-\gamma)^2} \cdot \left(\frac{1}{B_t} \cdot \omega_{it} \cdot \delta_{it}^{-\frac{2-\gamma}{1-\gamma}}\right)^2 + \frac{\gamma}{1-\gamma} \cdot \frac{2-\gamma}{1-\gamma} \cdot \frac{1}{B_t} \cdot \omega_t \cdot \delta_{it}^{-\frac{3-2\cdot\gamma}{1-\gamma}} \\
\frac{\partial^2(\gamma \cdot \ln B_t)}{\partial \delta_{it} \cdot \partial \delta_{jt}} &= -(\frac{\gamma}{1-\gamma})^2 \cdot \frac{1}{B_t^2} \cdot \omega_{it} \cdot \omega_{jt} \cdot \delta_{it}^{-\frac{2-\gamma}{1-\gamma}} \cdot \delta_{jt}^{-\frac{2-\gamma}{1-\gamma}}
\end{aligned}
$$

The local derivatives around the no dispersion point are thus given by

$$\begin{aligned}
\left.\frac{\partial \ln A_t}{\partial \tilde{\zeta}_{it}}\right|_{\zeta_{it}=\delta_{it}=1} &= \omega_{it} \\
\left.\frac{\partial \ln A_t}{\partial \delta_{it}}\right|_{\zeta_{it}=\delta_{it}=1} &= -\frac{\gamma}{1-\gamma}\cdot\omega_{it} \\
\left.\frac{\partial^2 \ln A_t}{(\partial \tilde{\zeta}_{it})^2}\right|_{\zeta_{it}=\delta_{it}=1} &= -\omega_{it}^2 \\
\left.\frac{\partial^2 \ln A_t}{(\partial \delta_{it})^2}\right|_{\zeta_{it}=\delta_{it}=1} &= -\left(\frac{\gamma}{1-\gamma}\cdot\omega_{it}\right)^2 + \frac{\gamma}{(1-\gamma)^2}\cdot\omega_{it} \\
\left.\frac{\partial^2 \ln A_t}{\partial \tilde{\zeta}_{it}\cdot\partial \delta_{it}}\right|_{\zeta_{it}=\delta_{it}=1} &= \frac{\gamma}{1-\gamma}\cdot(\omega_{it}^2-\omega_{it}) \\
\left.\frac{\partial(\gamma\cdot\ln B_t)}{\partial \delta_{it}}\right|_{\zeta_{it}=\delta_{it}=1} &= -\frac{\gamma}{1-\gamma}\cdot\omega_{it} \\
\left.\frac{\partial^2(\gamma\cdot\ln B_t)}{(\partial \delta_{it})^2}\right|_{\zeta_{it}=\delta_{it}=1} &= -\frac{\gamma}{(1-\gamma)^2}\cdot\omega_{it}^2 + \frac{\gamma}{1-\gamma}\cdot\frac{2-\gamma}{1-\gamma}\cdot\omega_t
\end{aligned}$$

The first-order approximation is given by

$$\ln \Xi_t \approx \ln 1 + \int_0^1 \left.\frac{\partial \ln A_t}{\partial \tilde{\zeta}_{it}}\right|_{\zeta_{it}=\delta_{it}=1} d\tilde{\zeta}_{it}\cdot di + \int_0^1 \left(\left.\frac{\partial \ln A_t}{\partial \tilde{\delta}_{it}}\right|_{\zeta_{it}=\delta_{it}=1} - \left.\frac{\partial(\gamma\cdot\ln B_t)}{\partial \tilde{\delta}_{it}}\right|_{\zeta_{it}=\delta_{it}=1}\right) d\delta_{it}\cdot di$$

It follows immediately that the first and last term are equal to 0. Finally, the second term is equal to 0 as well since, by definition of $\tilde{\zeta}_{it}$, $\int_0^1 \omega_{it}\cdot d\tilde{\zeta}_{it} = 0$.

Thus, we can focus on the second order approximation. Since the first order approximation is 0, it also follows that the aggregate terms $A_t$ and $B_t$ drop out in the second order approximation. It is thus given by

$$\begin{aligned}
\ln \Xi_t \approx& \frac{1}{2}\left(\int_0^1 \omega_{it}\cdot\left(\frac{\gamma}{(1-\gamma)^2} - \frac{\gamma}{1-\gamma}\cdot\frac{2-\gamma}{1-\gamma}\right)\cdot(d\ln\delta_{it})^2\cdot di - \int_0^1 \omega_{it}\cdot\frac{\gamma}{1-\gamma} d\ln\delta_{it}\cdot d\ln\tilde{\zeta}_{it} di\right) \\
=& -\frac{1}{2}\frac{\gamma}{1-\gamma}\left(\int_0^1 \omega_{it}\cdot(d\ln\delta_{it})^2 di + \int_0^1 \omega_{it}\cdot d\ln\delta_{it}\cdot d\ln\tilde{\zeta}_{it}\cdot di\right)
\end{aligned}$$

Defining the variance and covariance terms appropriately, we have the result in the Lemma.

□